A General Equilibrium Model of Accounting Standards and Enforcement

Hyun Hwang  
Carnegie Mellon University

Eunhee Kim  
City University of Hong Kong

Draft: November 2018

Abstract

We propose a general equilibrium model of accounting standards and regulatory enforcement, in which multiple firms prepare accounting reports (in accordance with the prevailing accounting standards) and compete for capital. In the model, capital is entrusted by a representative investor (investor) who allocates her endowment into consumption and savings based on publicly disclosed accounting reports. The investor enjoys the expected rate of return (cost of capital) which is determined by the demand for and supply of capital in the market. We show that firms tend to manipulate their reports as the wealth of the investor increases; the improvement of accounting standards and enforcement reduces the cost of capital, as do the aggregate investment and savings. Our findings suggest that the impact of accounting standards on the welfare of the investor and firms depends on the aggregate wealth of the economy.

Key Words: Accounting standards, enforcement, classifications manipulation, cost of capital, general equilibrium.
1. Introduction

Accounting standards are a set of financial reporting rules for firms in an economy. One of primary roles of accounting standards is to help capital providers better assess the viability of investment projects in multiple firms. As accounting standards are uniformly applied to multiple firms in an economy, the standards and their enforcement can affect macroeconomic variables (e.g., aggregate investment and the cost of capital) and the welfare of multiple stakeholders. While real-effects studies at the firm level suggest the implications of aggregate effects, the macroeconomic outcomes of accounting standards are relatively underexplored (Leuz and Wysocki, 2016). We fill this gap by proposing a model of accounting standards in which multiple firms prepare financial reports in accordance with accounting standards and regulatory enforcement to acquire capital from investors’ savings. In particular, we ask two questions. First, what are the impacts of accounting standards and their enforcement on macroeconomic outcomes (e.g., aggregate investment, the cost of capital, and investors’ consumption and savings) depending on the wealth of an economy? Second, how do accounting standards and enforcement affect the welfare of investors and firms?

We answer our research questions using a model with four features. First, every firm privately observes a signal that provides information about the probability of a project’s success, and prepares an accounting report consisting of one of two classifications: Better or Worse. The accounting report is dictated by accounting standards in the form of an official threshold. If a signal exceeds the official threshold, the firm reports Better; otherwise, the firm reports Worse. Second, given the governing accounting standards and the corresponding enforcement in an economy, some firms may attempt to manipulate their classifications at cost in order to report Better. Third, after observing the reports, the representative investor who has a consumption-smoothing motive allocates her endowment into consumption and savings, which form the supply of capital. Fourth, the cost of capital in the market is determined by equalizing the demand for and the supply of capital. In our paper, the cost of capital serves as the price in the capital market at which the quantity supplied by the investor’s savings is equal to the quantity demanded by firms. Our notion of cost of capital is similar in spirit with Sunder (2016, p 18) stating that the cost of capital is a rate of exchange as a price for a productive input.
To answer our first research question, we derive an equilibrium aggregate investment and cost of capital depending on the overall wealth of an economy. In a capital-deficient economy in which the wealth of the investor is low, we show that no classification manipulation exists in equilibrium. In addition, the cost of capital is high and the aggregate level of investment is low. Intuitively, a lower level of investor wealth implies a lower supply of capital. With the limited supply of capital, the cost of capital at which the supply meets the demand is likely to be expensive. In this case, firms that are sufficiently profitable can afford such expensive cost of capital, whereas less profitable firms choose not to manipulate their reports in the first place. We refer to this as self-selection in demand for capital. Thus, when the wealth of the investor is low, firms that have positive-NPV projects are not pooled with firms that have negative-NPV projects, suggesting that the market equilibrium cost of capital perfectly substitutes the role played by the accounting standards and enforcement.

However, in a capital-abundant economy in which the investor is endowed with enough wealth, classification manipulation always exists in equilibrium (Dye, 2002; Laux and Stocken, 2018). In this case, the cost of capital at which the supply meets the demand is lower, and the aggregate investment level is higher. With a sufficient wealth endowment, the investor is willing to save even for small returns, which lowers the cost of capital. Since firms can now borrow at the low cost of capital, less profitable firms are spurred to manipulate their reports, which leads to more aggregate investment. Thus, when the investor’s wealth is high, firms that have positive-NPV projects are pooled with firms that have negative-NPV projects. As a result, profitable firms subsidize less profitable firms that manipulate their classifications.

The immediate implication is that the role played by regulatory enforcement hinges on the aggregate wealth of an economy. When an economy lacks capital, imposing strong regulatory enforcement is irrelevant, as the cost of capital is sufficient to prevent less profitable firms from engaging in classification manipulation. However, as an economy becomes richer, the increasing wealth prompts classification manipulation, thereby increasing the demand for capital. Thus, strong enforcement has a non-trivial impact if an economy has enough wealth. This finding may help explain why capital-abundant countries, like the U.S., exhibit strong regulatory enforcement (Leuz and Wysocki, 2016).
To answer our second research question, we conduct a welfare analysis for the investor and different firms depending on accounting standards and their enforcement. Since the capital-lacking economy self-selects profitable firms via the high cost of capital instead of relying on regulatory enforcement, our welfare analysis is relevant to a capital-abundant economy. To compare the welfare of different stakeholders, we consider exogenous changes in accounting policies: lenient accounting standards (or enforcement) versus stringent accounting standard (or enforcement). When the existing accounting policy becomes more lenient (i.e., having a lower threshold or being easier to manipulate), it invites manipulation by even less profitable firms. We show that upon this change, the equilibrium cost of capital increases due to the increased demand for capital. While the increase in cost of capital makes the investor consume less, it can boost the investor’s welfare because of high returns on savings. The increase in the cost of capital as a result of lenient policy boosts the investor’s saving motive; thus, she is willing to cut back her consumption for better returns on savings. Moreover, the firms that manipulated their reports under the old policy are strictly better off under the new one because of the decreased manipulation costs, even though the cost of capital increases. The only stakeholders suffering from this lenient policy are profitable firms, due to the increase in the cost of capital and inefficient pooling.

On the other hand, when accounting policy becomes more stringent (having a higher threshold or being difficult to manipulate), classification manipulation apparently decreases. So, too, does the aggregate investment. We show that upon this policy change, the equilibrium cost of capital decreases due to the decrease in demand. The decrease in the cost of capital makes the investor consume more and save less, thereby reducing her welfare. Stringent policy change leads to an excess supply of capital, which lowers the return on savings the investor can enjoy. The decrease in the cost of capital makes savings less attractive, thereby lowering the investor’s saving motive. Essentially, strengthening accounting policy reduces the investor’s investment opportunity by lowering the demand for capital. Like the investor, the firms that manipulated under the old policy are hurt, since they no longer have access to capital. The only beneficiaries of the policy change are profitable firms: they are better off, since the cost of capital that they have to bear decreases and inefficient pooling is mitigated.
We deliver additional results that generate testable predictions of the cost of capital and classification manipulation with respect to the investor’s consumption-smoothing motive, the successful project outcome, accounting standards and enforcement. First, as the investor’s elasticity of substitution in consumption increases, the equilibrium cost of capital decreases, while more firms engage in classification manipulation. When the investor has a weak consumption-smoothing motive, it is easy to elicit savings (more supply), thus inviting more manipulation. Second, as the successful project outcome improves, more firms engage in classification manipulation, which in turn boosts the equilibrium cost of capital. With the shift in the successful outcome, classification manipulation becomes more profitable, thereby boosting the demand for capital. Lastly, as the accounting standards become more stringent or regulatory enforcement becomes stronger, fewer firms engage in classification manipulation and the equilibrium cost of capital decreases. The economic force that drives this result is the decrease in the demand for capital because of the increase in the manipulation cost.

Our paper builds on Dye (2002) and Laux and Stocken (2018), in which an accounting standard is modeled as an official threshold dichotomizing the state space to generate an accounting report. In these papers and ours, the role of an accounting standard and enforcement can be viewed as achieving efficient capital allocation. However, these papers focus on the demand side (by firms) and assume a perfectly elastic supply of capital. Ours, on the other hand, emphasizes the influence of accounting standards and enforcement on the demand for and supply of capital, which enables us to infer the cost of capital, the investor’s consumptions and savings, and aggregate investment in an economy.

Our paper also builds on Stiglitz and Weiss (1981), De Meza and Webb (1987), and Bernanke and Gertler (1990), which examine the impact of information asymmetry in the market for capital. However, the focus of these papers is the demand side in order to find an optimal means of financing (debt or equity) or to set an optimal policy by controlling the interest rate or strengthening net assets.¹ In our paper, the focus is on understanding how accounting standards and enforcement (which determine the degree of information asymmetry in the capital market) affects firms’ reporting behaviors (demand) and the investor’s intertemporal choices between

¹ Contrary to our model, in Stiglitz and Weiss (1981) and De Meza and Webb (1987), the capital supply is unlimited and perfectly elastic. In Bernanke and Gertler (1990), capital supply is enough to fund all projects available in the market.
consumption and savings (supply). Like us, Holmstrom and Tirole (1997) consider the supply side in the presence of moral hazard (of firms and banks) in the capital market; however their focus is again the demand side, and they do not address the investor’s intertemporal decisions.

There are also numerous studies on accounting standards including Dye and Sridhar (2008), Gao (2010), Bertomeu and Magee (2011), and Zhang (2013). By comparing rigid standards to flexible standards, Dye and Sridhar (2008) highlight the trade-off between reducing transaction manipulation (due to discretion exerted by managers) and suppressing substantive information (due to uniformity in treating heterogeneous transactions). Gao (2010) explores the links between disclosure quality, cost of capital, and investor welfare to show that reducing the cost of capital through better accounting quality may not always improve investor welfare due to a firm’s adjustments in its real investment decision. Bertomeu and Magee (2011) investigate the interaction between the business cycle and accounting regulation that is subject to political pressure. Zhang (2013) studies the impact of accounting standards on the cost of capital and welfare in an extended capital asset pricing model setting, and shows that high-quality accounting standards may generate a negative externality among firms with different levels of riskiness. We extend this literature by introducing the investor’s wealth, consumption and savings decisions, and by demonstrating that the macroeconomic consequences of accounting standards and enforcement differ depending on the investor’s wealth and the elasticity of substitution in consumption.

This paper is organized as follows. Section 2 describes a general equilibrium model of accounting standards. Section 3 includes the analysis of our model. Section 4 provides empirical implications. Section 5 concludes.

2. The Model

The model has two types of agents: a representative investor and a continuum of mass 1 of risk-neutral firms indexed by $i \in [0,1]$. There are three periods. In the first period, firms issue accounting reports in accordance with an accounting standard. In the second period, the investor makes consumption/saving decisions, and firms raise capital for their investment projects.
through the capital markets. In the final period, investment returns are realized and then distributed to firms and to the investor.

**Investment Project:** Firm $i$ has an investment project that requires a fixed capital investment $I$. In addition, firm $i$ incurs private cost $b > 0$ if the project is implemented. The constant cost parameter $b$ captures an outside option or opportunity cost in implementing the new production, which is borne by the firm (De Meza and Webb, 1988). If implemented, the project generates either cash flow $X$ with success probability $\theta_i$ or 0 with probability $1 - \theta_i$. Success probability $\theta_i$ is uniformly distributed over $[0,1]$, and $\theta_i$ is independent for $i \in [0,1]$. The unconditional NPV of each project is zero, that is, $E[\theta]X - I = 0$.

**Period 1 – Accounting Reports:** At period 1, firm $i$ privately observes $\theta_i$ and issues a report in accordance with an accounting standard. As in Dye (2002) and Laux and Stocken (2018), an accounting standard is modeled as a threshold $\theta_s$ such that the publicly observable report of firm $i$, $R_i$, is $R_i = W$ for all $\theta_i \in [0, \theta_s)$, and $R_i = B$ for all $\theta_i \in [\theta_s, 1]$, where $W$ and $B$ denote Worse and Better, respectively. With higher $\theta_s$, it is harder for firms to obtain report $B$, and higher $\theta_s$ can be understood as either a more stringent or a more conservative standard (Kwon et al., 2001; Laux and Stocken, 2018).

Our assumption of uniform distribution implies that, if all firms comply with the accounting standard, firms reporting $B$ have the expected conditional probability of success $E[\theta | \theta \geq \theta_s] = \frac{1+\theta_s}{2} > \frac{1}{2}$ for $\theta_s > 0$. Meanwhile, firms reporting $W$ have the expected conditional probability of success $E[\theta | \theta < \theta_s] = \frac{\theta_s}{2} < \frac{1}{2}$ for $\theta_s < 1$. Since we assume that the unconditional project payoff is zero, for any accounting standard $\theta_s \in (0,1)$, firms reporting $B$ have positive NPV, while firms reporting $W$ have negative NPV. Thus, the investor is willing to invest in the firm with report $B$, but not to inject capital into the firm with report $W$.

---

2 Firms may have to incur the cost (in addition to the necessary capital $I$) of planning, organizing, and executing the projects. Since firms will demand capital (to implement projects) only when the expected payoff, taking into account all the associated costs, is non-negative, the presence of cost $b$ makes different expected payoffs across firms, with different probabilities of success. Without cost $b$, every firm enjoys zero profit (by setting $D = X$) regardless of its realized probability of success, since $\theta_i$ is not observable and firms are competing for capital.

3 This assumption is meant to capture that the accounting report influences the investor’s savings decisions. See Laux and Stocken (2018) for a detailed discussion.
**Manipulation and Shadow Standard:** After privately observing $\theta_i$, firm $i$ can potentially manipulate its accounting report $R_i$. Specifically, we posit that firm $i$ with $\theta_i < \theta_s$ can obtain $R_i = B$ at private cost $\pi \times (\theta_s - \theta_i)$, where $\pi > 0$. High $\pi$ represents a strong enforcement regime. If firm $i$ with $\theta_i < \theta_s$ reports $R_i = B$, then firm $i$ is said to have violated the accounting standard. The lower the probability of success, the greater the manipulation cost for the firm to report $B$. If $\theta_i \geq \theta_s$, firm $i$ does not manipulate its accounting report $R_i = B$.

Let $\theta_T(r) \in (0,1)$ be the minimum probability for which firm $\theta_i \in [\theta_T(r),1]$ demands capital by reporting $B$. Hereafter, we refer to $\theta_T(r)$ as the shadow standard. Whether the shadow standard involves classifications manipulation or not depends on various factors, including the cost of capital $r$. Indeed, we will establish that the shadow standard is a function of the cost of capital $r$. For notational convenience, whenever there is no confusion, we omit the argument and use $\theta_T$.

The interpretation of the shadow standard $\theta_T$ is identical to those of Dye (2002) and Laux and Stocken (2018): $\theta_T$ is the effective partitioning of firms into Better projects and Worse projects. As in these two papers, when $\theta_T < \theta_s$, classification manipulation exists. However, in our model, it is also possible to have $\theta_T \geq \theta_s$ depending on the investor’s savings decisions. In this case, there is no classification manipulation, and some firms reporting $B$ may not be funded due to limited capital supply.

**Period 2 – The Clearing of the Capital Market:** The representative investor endowed with capital $\omega$ observes firms’ accounting reports and make consumption and savings decisions. The investor’s savings are used to fund projects at the rate of return $r$, which is determined by the supply of and demand for capital in the market. After observing accounting reports, the investor decides how much to consume now by $c \in [0, \omega]$ and how much to save by $s = \omega - c$, which gives them $s \times (1 + r)$ as a return at the end of the period.

Like Bernanke and Gertler (1990), we posit that this lending is carried by competitive financial intermediaries, such as banks, that do not incur any cost and earn zero profits in equilibrium. The function of financial intermediaries is to allocate the investor’s savings to firms

---

4 Here, $\pi$ also captures the probabilistic investigation by a regulatory body, in which case $\pi$ is interpreted as the probability of detection multiplied by any fine associated with its detected manipulation.
that demand capital. The cost of capital in the capital market is determined by equalizing the demand for capital to supply of capital that is freely aggregated through financial intermediaries.\(^5\)

We assume that the investor’s preference is quasi-linear, with two additively separable components: a strictly increasing and concave utility for current consumption \(c\) and linear utility from savings \(s\) as follows:

\[
U(c, s) \equiv u(c) + s \times (1 + r),
\]

where \(u(c) = \frac{1}{1-\alpha} c^{1-\alpha}\) is a concave increasing function of \(c\). The quasi-linear utility assumption is useful to capture that the investor has an imperfectly elastic consumption-smoothing motive, while maintaining risk neutrality with respect to expected returns (Tirole, 2006; Dicks and Fulghieri, 2018).\(^6\) In particular, the parameter \(1/\alpha\) represents the elasticity of intertemporal substitution in consumption and savings.\(^7\) High \(1/\alpha\) implies that the substitution effect is high: consumption change is very sensitive to interest rate change. Introducing the elasticity of substitution provides an economic rationale for consumption smoothing (Christensen, de la Rosa, and Feltham, 2010). Our assumption regarding the investor’s preference thus allows us to focus on the role of the investor’s consumption-smoothing motive in determining the available capital supply while abstracting away from risk premiums. The consumption and savings decisions depend on the savings return \(r\). Since the investor’s required rate of return is the cost for firms to implement their projects, we use the cost of capital, savings return, and interest rate interchangeably.

Capital is invested in a project in firm \(i\) through a standard debt contract with fixed repayment \(D \in [0, X]\) upon the realization of \(X\). If the project fails, the repayment is zero.\(^8\) The investor cannot distinguish the probability of success among firms reporting \(B\). Accordingly, the

---

\(^5\) This modeling choice is similar to those made by Stiglitz and Weiss (1981), De Meza and Webb (1987), and Holmstrom and Tirole (1997). These papers (except Bernanke and Gertler, 1990) emphasize the incentive of financial intermediaries (i.e., banks). In our paper, however, the main departure is the emphasis on the investor’s allocation decisions—consumption and savings—as influenced by accounting standards and enforcement.

\(^6\) The same formulation for investors’ preferences is used in Dicks and Fulghieri (2018) to distinguish the liquidity shock-driven consumption in early date from investment payoff at later date.

\(^7\) Specifically, by using the marginal rate of substitution, we have \(\frac{s}{c} = \left(\frac{u'(c)}{1+r}\right)^{\frac{1}{\alpha}}\), where \(1+r\) denotes the marginal utility of savings (Mas-Colell, Whinston, and Green, 1995).

\(^8\) As in Laux and Stocken (2018), this contract can also be interpreted as an equity contract where each firm sells a fraction \(D/X\) of the firm.
expected repayment must be equal to the expected repayment from any funded firm with report $B$. Let $\hat{\theta}_T$ denote the shadow standard the investor believes. That is, the investor believes that the probability of success of the least profitable funded firm is $\hat{\theta}_T$. The investor does not consider the private cost $b$ for firm $i$ when making the investment decision, because this cost is borne by the firm. Therefore, repayment $D$ must satisfy:

$$E[\theta|\theta \geq \hat{\theta}_T] \times D = (1 + r) \times I.$$  

As discussed above, the probability of the least profitable funded firm $\hat{\theta}_T$ can be greater than or equal to $\theta_s$ if there is no classifications manipulation in equilibrium. Or, $\hat{\theta}_T$ can be less than $\theta_s$ if there are some firms manipulating their classifications to obtain capital.

Each firm takes the fixed repayment $D \in [0, X]$ as given. If the expected payoff of implementing the project is negative, the firm chooses not to implement the project and obtains zero payoff. Thus, the expected payoff to firm $i$ on a project with report $B$ is:

$$\begin{cases} 
\max\{\theta_i(X - D) - b, 0\} & \text{if } \theta_i \geq \theta_s, \\
\max\{\theta_i(X - D) - b - \pi \times (\theta_s - \theta_i), 0\} & \text{if } \theta_i < \theta_s,
\end{cases}$$

(2)

The capital market clears when the aggregate corporate investment is equal to the investor’s savings. That is,

$$\int_{\theta \geq \theta_T(r)} \theta dF(\theta) = s(r) = \omega - c(r),$$

(3)

where $c(r)$ and $s(r)$ denote, respectively, the investor’s consumption and savings at the interest rate $r$.

**Period 3 – Outcomes:** After the capital is injected, the funded project generates an outcome. Firm $i$ receives a payoff—specified in (2)—if the project is funded and succeeds. With $\theta_T$ being the probability of success of the least profitable funded firm, the aggregate outcome $\int_{\theta \geq \theta_T} \theta dF(\theta)$ yields a return for the investor, who obtains the payoff specified in (1). Since the shadow standard $\theta_T$ is located at the margin that determines the range of funded firms, we call it the marginal firm, and use the marginal firm and the shadow standard interchangeably. Technically, the marginal firm is determined by a zero-profit condition of the least profitable firm in the payoff expression (2).
An equilibrium requires that firms report their classifications to maximize their payoff (2) for a given $r$, that the investor makes consumption and savings decisions to maximize the payoff (1) for a given $r$, and that the cost of capital $r$ is determined by the market clearing condition (3). Formally, our equilibrium is defined as follows:

**The Definition of Equilibrium:** For a given $(\omega, \alpha, \theta_s, \pi, X, b)$, an equilibrium consists of a tuple $(\theta_T(r^*), c(r^*), D(r^*), r^*)$ such that:

(i) Every firm reports $B$ if $\theta \geq \min\{\theta_T(r^*), \theta_s\}$, or reports $W$ if $\theta < \min\{\theta_T(r^*), \theta_s\}$,

(ii) The investor chooses consumption and savings to maximize her payoff given $r^*$:

$$c(r^*) \in \arg\max_c u(c) + (1 + r^*)(\omega - c).$$

(iii) The expected repayment ensures the investor breaks even at the interest rate $r^*$, and the investor correctly anticipates $\theta_T(r^*)$:

$$E[\theta|\theta \geq \hat{\theta}_T]D(r^*) = (1 + r^*)I, \quad \hat{\theta}_T = \theta_T(r^*).$$

(iv) The marginal firm makes zero profit:

$$\theta_T(r^*) \times (X - D(r^*)) - \max\{0, \pi(\theta_s - \theta_T(r^*))\} = b.$$

(v) The market clears:

$$s(r^*) = \int_{\theta \geq \theta_T(r^*)} l dF(\theta), \text{ where } s(r^*) = \omega - c(r^*).$$

Figure 1 summarizes the timeline.

<table>
<thead>
<tr>
<th>$t = 1$</th>
<th>$t = 2$</th>
<th>$t = 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firms privately observe $\theta$ and report $B$ or $W$.</td>
<td>The investor makes consumption and savings decisions; the market clears.</td>
<td>Project outcomes and payoffs are realized.</td>
</tr>
</tbody>
</table>

Figure 1: Timeline
3. Equilibrium Analysis

First-Best To derive a financial reporting equilibrium, it is useful to derive an equilibrium using the first-best case as a benchmark. The first-best case is the full information competitive equilibrium, wherein firms’ success probabilities are observable. In this case, the repayment $D$ can be tied to each firm’s success probability $\theta$. If the investor’s endowment is sufficiently high (in a capital-rich economy), then it is efficient to fund all positive-NPV projects that do not yield negative payoff to firms at the cost of capital $r = 0$. This depends on the opportunity cost $b$. We posit that $b$ is sufficiently small $\frac{b}{X} < \frac{1}{2}$ for there to always be firms that demand capital.

On the other hand, if the investor’s endowment is not sufficient (in a capital-deficient economy), then it is efficient to fund only those sufficiently profitable firms at the cost of capital $r > 0$ at which the market clears. The following lemma finds the equilibrium when $\theta$ is observable, which depends on the endowment level. We use the superscript $FB$ to denote the first-best.

**Lemma 1.** (First-Best) Let $\theta_{FB}$ denote the success probability of the marginal firm, $r_{FB}$ denote the cost of capital, and $\omega^0 \equiv \frac{X}{4} - \frac{b}{2} + 1$. The first-best equilibrium is expressed as follows.

i) The investor’s choices of consumption $c_{FB}$ and savings $s_{FB}$ are $c_{FB} = \frac{1}{1 + r_{FB}} \frac{1}{a}$ and

$$s_{FB} = \omega - \frac{b}{1 + r_{FB}}.$$

ii) The repayment of firm $\theta$ is $D(\theta) = \frac{1 + r_{FB}}{2\theta} X$.

iii) Firms obtain capital if and only if their success probabilities are greater than or equal to $\theta_{FB} = \frac{1 + r_{FB}}{2} + \frac{b}{X}$

iv) If $\omega \geq \omega^0$, then $r_{FB} = 0$. If $\omega < \omega^0$, the cost of capital $r_{FB} > 0$ is uniquely determined by the market clearing condition:

$$\frac{X(1 - r_{FB})}{4} - \frac{b}{2} + 1 = \omega - \frac{1}{1 + r_{FB}} \frac{1}{a}.$$
Here, the success probability of the marginal firm in the first-best is greater than 1/2 due to the firm’s opportunity cost. Since the probability of success is observable, there is neither pooling across firms nor cross-subsidization: every firm that demands capital bears the same expected repayment (i.e., financing cost). Profitable firms have lower repayment $D(\theta)$ since they will repay with higher probability, whereas less profitable firms have greater repayment since they will repay with lower probability.

It is worth discussing that the first-best cost of capital can vary depending on the supply of capital: a strictly positive cost of capital might be required to elicit savings. The cost of capital is positive, not because of information friction, but because of lack of capital supply. Thus, interpreting $r^{FB} > 0$ must be based on the required rate of return to elicit savings.

**The Other Extreme Case** Another benchmark could be a situation in which a firm’s success probability is not observable, and there is no way for that firm to communicate the realized probability. We show in the appendix that this yields the maximum level of aggregate investment, and the most inefficient pooling and cross-subsidization. We will discuss this point more precisely after deriving a financial reporting equilibrium.

To derive a financial reporting equilibrium, we first derive a demand curve and supply curve for a given cost of capital $r$. After we characterize the equilibrium, we conduct numerous comparative statics to understand the behaviors of equilibrium consumption, savings, classification manipulation, and the cost of capital.

### 3.1 Demand for Capital

We start from a market demand function for a given interest rate $r$. Since $\theta$ is not observable, the expected repayment is determined by the expectation of $\theta$ conditional on the report $B$. Recall that $\theta_T(r)$ denotes the success probability of the marginal firm that is funded last. Since every firm $\theta \geq \theta_T(r)$ reports $B$, we have

$$E[\theta|\theta \geq \theta_T(r)]D = (1 + r)I \Rightarrow D = \frac{1 + r}{1 + \theta_T(r)}X,$$


where we use $E[\theta]X = I, I = \frac{X}{2}$. First, consider a firm with $\theta \geq \theta_s$ that does not have to manipulate its report. Firm $\theta$’s payoff at the repayment $D$ is then:

$$
\theta(X - D) - b = \theta \left( X - \frac{1 + r}{1 + \theta_T(r)}X \right) - b \\
= \frac{\theta X}{1 + \theta_T(r)}(\theta_T(r) - r) - b. \quad (4)
$$

Firm $\theta \geq \theta_s$ demands capital as long as the payoff (4) is greater than or equal to zero.

Consider a firm with $\theta < \theta_s$ which must incur the cost of manipulation in order to report $B$. As in (4), the manipulating firm $\theta$’s payoff is given by:

$$
\theta(X - D) - b - \pi(\theta_s - \theta) = \frac{\theta X}{1 + \theta_T(r)}(\theta_T(r) - r) - b - \pi(\theta_s - \theta). \quad (5)
$$

Firm $\theta < \theta_s$ demands capital by manipulating its classifications as long as its net payoff (5) is greater than or equal to zero.

Since the marginal firm $\theta_T(r)$ is the least profitable funded firm, it earns zero profit. The marginal firm’s payoff from investment is determined by its success probability $\theta$, cost of capital $r$, opportunity cost $b$, and investment earnings $X$, as well as the accounting standard $\theta_s$ and enforcement level $\pi$. Depending on the cost of capital $r$ and opportunity cost $b$, the marginal firm is characterized by either (4) or (5). If the marginal firm is determined by (4) due to high cost of capital $r$, then the marginal firm does not manipulate its classification. If the marginal firm is determined by (5) due to low cost of capital $r$, then the marginal firm does manipulate its classification. The following lemma characterizes the success probability of the marginal firm.

**Lemma 2.** (Demand for capital) Let $\theta_s$ and $r$ be given. The market demand for capital is characterized as the marginal firm $\theta_T(r)$, such that:

$$
\theta_T(r) = \begin{cases} 
 v(r) + \sqrt{v(r)^2 + \frac{b}{X}} & \text{if } Q^D \leq (1 - \theta_s)I, \\
 v_T(r) + \sqrt{v_T(r)^2 + \frac{b + \pi \theta_s}{X + \pi}} & \text{if } Q^D > (1 - \theta_s)I,
\end{cases}
$$
\[ v(r) = \frac{b + rX}{2X}, \quad v_T(r) = \frac{b + rX - \pi\theta_s}{2(1 + X)}, \text{ and } Q^D \text{ denotes the aggregate demand for capital.} \]

Notice that as \( r \) increases, the success probability of marginal firm that demands capital increases as well. Intuitively, as the cost of capital becomes more expensive, the firm that can afford such expensive capital must be more profitable. Since the market demand is \( Q^D = (1 - \theta_T(r))I \), the demand decreases as the cost of capital increases. Due to manipulation cost, however, the demand from the manipulating firms is more sensitive to the increase in the cost of capital than that of the non-manipulating firms.

In our model, as noted before, the marginal firm represents the effective partitioning of firms into funded and unfunded firms. If \( \theta_T(r) \geq \theta_s \) is the marginal firm, then, there may exist non-manipulating firms that report \( B \) but still do not obtain capital. This is because these firms choose not to implement projects due to their low probability of success relative to the cost of capital (self-selection). On the other hand, if \( \theta_T(r) < \theta_s \) is the marginal firm, then there exist some firms that report \( B \) by incurring manipulation costs in order to obtain capital. These firms \( \theta \in [\theta_T(r), \theta_s) \) are willing to incur such costs because, even though their probability of success is low, implementing projects is still profitable relative to cost of capital. In the following section, we derive a supply of capital.

3.2 Supply of Capital

Since the investor’s utility is concave, her optimal consumption and savings choices are determined by the first order condition:

\[ \frac{dU}{dc} = c^{-\alpha} - (1 + r) = 0 \Rightarrow c = \frac{1}{1 + r}. \]

As \( r \) increases, consumption decreases (and savings increase). For any \( \alpha \in (0,1) \), the maximum consumption level is 1 when \( r = 0 \). Given \( r \geq 0 \), if \( \omega \leq \frac{1}{1 + r} \), then the investor’s utility-maximizing decision is to consume all of her endowment without savings. To make the investor’s capital allocation decision nontrivial, we assume that \( \omega \) is not so small that there exist non-zero savings to be allocated.
**Assumption.** \( \omega > 1. \) **Investor’s savings are strictly positive for any** \( r \geq 0. \)

As long as \( \omega > 1, \) the investor’s consumption choice is fully characterized by the savings return \( r. \) That is, in our model, the investor’s consumption choice does not have an income (endowment) effect, but it has a price effect through \( r. \) As the savings return \( r \) increases, the investor consumes less and saves more. Thus, the investor’s savings can be written as a function of interest rate \( r: \)

\[
s(r) = \omega - \frac{1}{a + r} = Q^S,
\]

where \( Q^S \) denotes the supply of capital. As \( r \) increases, the market supply increases.

The following lemma summarizes the discussion.

**Lemma 3.** (Supply of Capital) **Let** \( r \) **be given. The supply of capital is expressed as:**

\[
s(r) = \omega - \frac{1}{a + r}.
\]

3.3 **Financial Reporting Equilibrium**

With the demand for and supply of capital, we now derive a financial reporting equilibrium that depends on the wealth of an economy. Proposition 1 finds a financial reporting equilibrium depending on the investor’s endowment.

**Proposition 1.** (Financial Reporting Equilibrium) **Let** \( \theta_T(r) \) **denote the success probability of the marginal firm expressed in Lemma 2 and** \( r^* \) **denote the equilibrium cost of capital. The investor’s choices of consumption** \( c(r^*) \) **and savings** \( s(r^*) \) **are:**

\[
c(r^*) = \frac{1}{a + r^*} \quad \text{and} \quad s(r^*) = \omega - \frac{1}{a + r^*}.
\]
\( \frac{1}{1+r^*} \), and the repayment of firms that demand capital is \( D(r^*) = \frac{1+r^*}{1+\theta_T(r^*)} X \). Firm \( \theta \) obtains capital if and only if its success probability is greater than or equal to \( \theta_T(r^*) \) where

\begin{enumerate}
  \item[i)] if \( \omega \geq \overline{\omega} \), then \( r^* = 0 \), and \( \theta_T(r^*) < \theta_s \), and the firms manipulate their classifications for all \( \theta \in [\theta_T(r^*), \theta_s) \).
  \item[ii)] If \( \omega < \omega < \overline{\omega} \), then \( r^* > 0 \) and \( \theta_T(r^*) < \theta_s \), and the firms manipulate their classifications for all \( \theta \in [\theta_T(r^*), \theta_s) \).
  \item[iii)] If \( \omega < \omega \), then \( r^* > 0 \) and \( \theta_T(r^*) > \theta_s \), and every firm fully complies with the standard.
\end{enumerate}

Where \( r^* > 0 \) is uniquely determined by the market clearing condition,

\[
(1 - \theta_T(r^*))I = \omega - \frac{1}{1+r^*},
\]

and the expressions of \( \omega \) and \( \overline{\omega} \) are presented in the appendix A.

Figure 2 depicts an example of an equilibrium cost of capital. When the endowment is too high (\( \omega \geq \overline{\omega} \)), the investor is willing to save even at a zero interest rate. In this case, firm \( \theta \in [\theta_T(0), \theta_s) \) can have access to capital by manipulating its classifications. Notice that firm \( \theta < \theta_T(0) \) does not engage in classification manipulation not because the supply is insufficient nor the cost of capital is expensive, but simply because it is not profitable to manipulate due to the manipulation cost \( \pi(\theta_s - \theta) \) and opportunity cost \( b \). This finding provides a rationale for why capital-abundant countries, like the U.S., exhibit stronger enforcement than capital-deficient countries. When the endowment is in the intermediate range (\( \omega < \omega < \overline{\omega} \)), savings may be substantial enough that some firms still find it profitable to manipulate. The required interest rate differs depending on the investor’s wealth, thereby changing the equilibrium level of classification manipulation.
Figure 2. Equilibrium Cost of Capital and Classification Manipulation

Figure 2 depicts an equilibrium cost of capital and classification manipulation, which depend on the wealth of the investor, for the following parameter values: $\theta_s = 0.5, \pi = 0.3, I = 1.5, b = 0.1, a = 0.5, \omega = 1.1$ (for the low wealth) and 1.4 (for the high wealth). The X-axis denotes the quantity (demand and supply) and the Y-axis denotes the cost of capital. The downward-sloping line is the inverse demand curve; the dashed portion represents the capital demand from manipulating firms. The two upward-sloping lines are the inverse supply curves. When investor wealth is low, no classification manipulation exists, whereas classifications manipulation exists when investor wealth is high. The equilibrium cost of capital is higher when the investor wealth is low than when the investor wealth is high.

On the other hand, when the investor’s endowment is sufficiently small ($\omega \leq \omega$), then there will be no firms manipulating their classifications $\theta_T(r^*) \geq \theta_s$. In a capital-deficient economy, the market interest rate is high enough that implementing less profitable projects ($\theta < \theta_s$) by incurring manipulation costs yields negative payoffs for firms that do so. This finding implies that in capital-deficient countries, the market interest rate is so effective that it perfectly substitutes for the role of enforcement: even without enforcement, no firm wants to manipulate. This finding lends new insight into how self-selection in capital demand may get rid of classification manipulation due to an imperfectly elastic and limited capital supply.

**Proposition 2.** (Cost of Capital as Substitute for Enforcement) *The equilibrium cost of capital $r^*$ screens less profitable firms. In particular, if $\theta_T(r^*) \geq \theta_s$, then classification manipulation does*
not exist in the equilibrium due to the high cost of capital $r^*$. That is, the equilibrium cost of capital $r^*$ substitutes for regulatory enforcement.

Compared to the first-best case, when $\theta$ is not observable, the investor cannot differentiate between firms that report $B$. As shown in section 3.1, the pooling repayment is $D = \frac{1+r}{1+\theta_T(r)} X$, where $\theta_T(r)$ denotes the marginal firm that is funded. In this case, more profitable firms borrow the same level of capital at rates relatively more expensive than less profitable firms. That is, there is cross-subsidization (Stiglitz and Weiss, 1981; De Meza and Webb, 1987). The extent of pooling across firms in equilibrium depends on the investor’s endowment $\omega$, which determines the cost of capital. In Proposition 3, we show that some negative-NPV projects might have access to capital due to cross-subsidization.

**Proposition 3.** (Cross-subsidization) When $\theta$ is not observable, more profitable firms subsidize less profitable firms. As the success probability of the marginal firm decreases, the cross-subsidizing repayment $D(r) = \frac{1+r}{1+\theta_T(r)} X$ increases. Given that $\frac{b}{X}$ is sufficiently small, there exists $r > r' > 0$ such that the aggregate investment is greater than the first best:

i) If the market-clearing interest rate is $r' < r \leq r$, then firm $\theta \in [\theta_T(r), \theta^{FB})$, where $\theta_T(r) \geq \theta_s$, obtains capital without manipulation.

ii) If the market-clearing interest rate is $r \leq r'$, then firm $\theta \in [\theta_T(r), \theta^{FB})$, where $\theta_T(r) < \theta_s$, obtains capital with manipulation; i.e., some negative-NPV projects are funded.

The expressions for $r$ and $r'$ and the conditions for $\frac{b}{X}$ are presented in appendix A.

As less profitable firms have more access to capital, the expected probability of success (the conditional probability that the investor will receive the repayment $D(r)$) becomes lower. Thus, firms must pay higher $D(r)$ to the investor in order to compensate for the lower
probability of success. Since the expected repayment of firm $\theta$ is $\theta \times D(r)$, profitable firms pay relatively more than less profitable firms. While every firm that demands capital faces the same level of cost of capital (as a market clearing rate), the expected repayment differs depending on its true success probability. The presence of cross-subsidization may lead to the success probability of the marginal firm being lower than the first-best; in such a case, some firms that cannot obtain capital under the first-best now have access to it. Cross-subsidization is therefore the key economic force. First, on the supply side, it can still be profitable for the investor to put her wealth into savings because of the increase in savings returns. Second, on the demand side, every firm that demands capital faces the same repayment, since the true success probability is unobservable. Thus, less profitable firms pay the repayment less often, which lowers their expected repayment.\(^9\) This makes it possible for some firms (even those with negative-NPV projects) to receive capital in equilibrium.

Turning to our discussion in the first best benchmark, our results in Proposition 3 provide an economic rationale for accounting standards and enforcement. Without accounting standards or enforcement, there can be too much investment in which capital is injected into even some negative-NPV projects. Less profitable firms demand capital due to cross-subsidization, and profitable firms bear increased financing costs. The value of properly established accounting standards and enforcement is to mitigate potential inefficiency while providing investment opportunities for both firms and the investor.

To summarize, the investor’s wealth endowment, consumption-smoothing motive, accounting standards and enforcement determine the equilibrium level of classification manipulation. Explaining how the changes in these variables affect the equilibrium cost of capital requires comparative statics analysis, which we will conduct in the next section.

4. Comparative Statics

4.1. Endowment $\omega$, Elasticity $a$, Productivity $X$, Accounting Standard $\theta_s$, Enforcement $\pi$

\(^9\) Indeed, due to cross-subsidization, every firm wants to obtain capital ex post. The logic is similar in spirit to that of limited liability: firms have nothing to lose when the outcome is low. What prevents them from demanding capital is the presence of manipulation cost and opportunity cost by $b$. 
In this section, we analyze how the equilibrium cost of capital and the shadow standard are affected by the characteristics of the investor, production technology, and accounting policy variables. In particular, we investigate how the change in endowment ($\omega$), the elasticity of substitution ($a$), productivity level ($X$), the accounting standard ($\theta_s$), or enforcement ($\pi$) affects the equilibrium cost of capital. Notice that capital supply changes as the investor endowment and income-smoothing motive, as well as the cost of capital ($\omega, a,$, and $r$), change. Similarly, capital demand changes as the accounting standards, enforcement level, productivity, and cost of capital ($\theta_s, \pi, X$, and $r$) change. When conducting comparative statics, it is useful to denote $Q^S(\omega, a, r)$ and $Q^D(\theta_s, \pi, X, r)$, respectively, as the supply and demand functions. In equilibrium, we have:

$$Q^S(\omega, a, r) = Q^D(\theta_s, \pi, X, r).$$

Since the cost of capital $r$ is endogenous, we use the implicit function theorem to conduct comparative statics for the cost of capital $r$. Proposition 4 summarizes the result.

**Proposition 4.** (Comparative Statics of the Cost of Capital) As the endowment $\omega$ increases, the elasticity of substitution in income increases, and the cost of capital decreases. As the production size becomes greater, the cost of capital increases. As the accounting standard $\theta_s$ increases, or the enforcement level $\pi$ increases, the cost of capital decreases. That is,

$$\frac{\partial r^*}{\partial \omega} < 0, \quad \frac{\partial r^*}{\partial a} > 0,$$

$$\frac{\partial r^*}{\partial X} > 0, \quad \frac{\partial r^*}{\partial \theta_s} < 0, \quad \frac{\partial r^*}{\partial \pi} < 0.$$

Intuitively, as the investor is endowed with more capital, the required interest rate decreases (the supply effect). This part is intuitive, as the investor is more willing to save when she has enough to allow consumption. Recall that the parameter $1/a$ measures the elasticity of substitution between consumption and savings returns. The increase in $a$ lessens the elasticity smaller, thereby being less sensitive to interest rate: the consumption motive is stronger as $a$ increases. Therefore, to induce savings, the required interest rate has to increase: $\frac{\partial r^*}{\partial a} > 0$. This
suggests that when the investor has strong consumption motive, it is relatively difficult to induce the investor to save, thereby making capital more expensive.

Moreover, as the project outcome $X$ increases, the equilibrium cost of capital increases. The intuition is because as implementing investment is more profitable for firms, more of them will demand capital, which in turn increases the cost of capital (demand effect). In a similar way, as the accounting standard $\theta_s$ becomes more conservative, the required cost of capital decreases. We show in the proof for Proposition 4 that this is not because there is less inefficient pooling with the conservative standard, but because the overall demand decreases. In our fixed investment model, strengthening accounting standards by raising $\theta_s$ reduces the aggregate investment (demand for capital). Upon this increase in standards, profitable firms enjoy a reduction in expected repayments due to the decrease in pooling. This finding suggests that having strong accounting standards makes those profitable firms that demand capital regardless of the standard better off, since not many firms can demand capital. Similarly, the demand effect is in force for the enforcement $\pi$: the increase in enforcement $\pi$ reduces the demand for funding from classification-manipulating firms. Reducing the demand from manipulating firms in turn reduces the cost of capital. As in the case for $\theta_s$, strengthening the enforcement level unambiguously makes profitable firms better off. We provide numerical examples to illustrate Proposition 4 in appendix B.

Now, we investigate how the shadow standard (the success probability of the marginal firm) changes with respect to accounting standards, enforcement, and endowment. While the direct dependency of these exogenous variables appears to be clear, there is also an indirect dependency through the cost of capital. For instance, the increase in enforcement may discourage classification manipulation (thus, leading to an increase in $\theta_T(r)$), while the reduction in the cost of capital $\frac{\partial r^*}{\partial \pi} < 0$ (Proposition 4) may restore the manipulation incentive. The following proposition summarizes the aggregate effect of these exogenous variables on the shadow standard. Note that the comparative statics of the shadow standard with respect to the two accounting variables $\theta_s$ and $\pi$ are only for $\omega \geq \bar{\omega}$, as otherwise the marginal firm is independent of these two accounting variables.
Proposition 4. (Shadow Standard $\theta_T(r^*)$) As the aggregate wealth of the economy increases, the elasticity of substitution in consumption increases, or project earnings becomes greater, more firms engage in classification manipulation. On the other hand, as the accounting standard becomes conservative and/or enforcement becomes stronger, fewer firms engage in classification manipulation. That is,

$$\frac{d\theta_T(r^*)}{d\omega} < 0, \quad \frac{d\theta_T(r^*)}{da} > 0,$$

$$\frac{d\theta_T(r^*)}{dX} < 0, \quad \frac{d\theta_T(r^*)}{d\theta_s} > 0, \quad \frac{d\theta_T(r^*)}{d\pi} > 0.$$ 

It turns out that the total effect of variables, $\omega, X, \theta_s,$ and $\pi,$ on the shadow standard is determined by the direct dependency: the indirect dependency created through the cost of capital is dominated. In contrast, the total effect of the investor’s elasticity of substitution on the shadow standard is fully characterized by the indirect dependency through the cost of capital. First, the increase in capital endowment boosts classification manipulation motives, as the cost of capital will be lower. Interestingly, firms’ incentive to manipulate their reports is also influenced by the investor’s elasticity of substitution in consumption. While the investor’s income-smoothing motive (parameter $a$) does not directly factor into in firms’ expected payoffs, it does affect those payoffs through the cost of capital $r^*$. Since the cost of capital increase as the investor’s elasticity of substitution decreases (i.e., there is an increase in $a$), fewer firms engage in classification manipulation.

As project earnings increase, the marginal benefit of implementing a project also increases. Therefore, more firms engage in classifications manipulation. Even though the increase in demand boosts the cost of capital, the net effect is always negative, and more firms manipulate: if fewer firms manipulate upon the increase in $X$, then the decrease in demand lowers the cost of capital, which implies that the marginal firm enjoys a strictly positive payoff. This elicits manipulation from less profitable firms.

Lastly, the increase in $\theta_s$ or $\pi$ discourages a manipulation motive, even though such changes lower the cost of capital (Proposition 3). The intuition is again related to the demand
effect. If more firms manipulate (creating a decrease in the success probability of the marginal firm), the increased demand must be supported by an increase in supply, which implies an increase in interest rate. Because the marginal firm that earns zero profit before the change cannot bear the increased classification manipulation cost (combined with the increase in the cost of capital due to more demand), the success probability of that firm cannot decline.

Proposition 4 again provides the rationale for why capital abundant countries need either stringent standard or strong enforcement, as expressed in Proposition 3. The difference from Proposition 3 is that Proposition 4 directly investigates the impact of accounting standards and enforcement based on a general equilibrium interaction. Moreover, Proposition 4 implies that the necessity of such accounting regulations (both $\theta_s$ and $\pi$) is influenced by the investor’s income-smoothing motive (elasticity of substitution) and the magnitude of project earnings.

Our comparative statics results, which are based on a general equilibrium interaction provide a new insight that differs from a partial equilibrium analysis. The natural follow-up question is to inquire about welfare implications. Since our objective is to infer policy implications using a general equilibrium interaction, in the next subsection, we investigate how firms’ and the investor’s welfare shifts as a result of changes in standards or enforcement.

4.2. Welfare Consequences of Changes in Accounting Standards and Enforcement

In this section, we investigate externalities among different stakeholders. In particular, we explore how the welfare of investors and firms changes with a change in accounting standards and enforcement. When the investor’s endowment is sufficiently high ($\omega < \omega < \bar{\omega}$) so that some firms attempt to manipulate their accounting reports while the investor is enjoying positive savings returns, the change in these two accounting policy variables affect the equilibrium cost of capital and aggregate investment (thus, the investor’s and firms’ welfare). We consider two changes in the standard and enforcement: 1) lenient accounting, in which the standard becomes lower or the enforcement level becomes weaker (lowering $\theta_s$ or $\pi$); and 2) stringent accounting, in which the standard becomes greater or the enforcement level becomes stronger (increasing $\theta_s$ or $\pi$). These two cases are meant to reduce the deadweight loss from the manipulation cost and
inefficient cross-subsidization. The following proposition finds how the changes in $\theta_s$ or $\pi$ affect different stakeholders’ payoffs.

**Proposition 5.** (Lenient Accounting Versus Stringent Accounting) Let $\underline{\omega} < \omega < \overline{\omega}$. Lenient accounting (lowering $\theta_s$ or $\pi$) may lead to 1) a decrease in consumption and increase in savings, and 2) an increase in investor welfare. This change makes 3) manipulating firms better off, but 4) profitable firms worse off. On the other hand, stringent accounting (increasing $\theta_s$ or $\pi$) leads to 1) an increase in consumption and a decrease in savings, thereby 2) lowering investor welfare. This change 3) causes a dropout by manipulating firms, thus lowering their welfare, whereas 4) benefiting profitable firms only.

When capital supply is enough that even some manipulating firms are funded, lenient accounting leads to excess demand by lowering classification manipulation costs. This excess demand boosts the market clearing interest rate, which makes savings more attractive. Upon this change, the investor becomes willing to cut back her current consumption level so as to save. This surely increases the investor’s welfare due to the increased savings returns. While the cost of capital increases due to this change, the firms that manipulated before the change in accounting policy are better off because they bear less manipulation cost. In particular, the change in standards generates another shadow standard. In this case, the newly-funded firms that manipulate their reports are better off, because they have access to funding. However, the profitable firms that did not have to manipulate before the change in accounting policy are hurt by this change, because 1) there is more pooling (exacerbated cross-subsidization), and 2) the excess demand boosts the cost of capital $r$, thereby pushing up the equilibrium repayment $D(r)$, and thus the expected financing cost $\theta \times D(r)$.

On the other hand, stringent accounting leads to excess supply. This excess supply, in turn, lowers the market-clearing interest rate, which makes savings less attractive. Upon this change, the investor becomes less willing to save and increases consumption. Since the investor’s welfare is an increasing function of $r$, a cutback in savings due to low $r$ lowers the investor’s welfare. This result is something of surprise to us, since we expected that increasing
the average success probability of funded firms would benefit the investor. The intuition for this result is related to the investor’s motive for savings. Since the investor is endowed with enough capital to offer a supply and they require an interest rate high enough to secure a return on savings, creating excess supply (by reducing demand) essentially makes the investor worse off because they cannot ask the same interest rate any more.

Meanwhile, the firms that have no longer access to capital under a stringent accounting policy are hurt. These include both 1) the firms that manipulated their reports before the change and 2) the firms with probabilities of success that exceed the old accounting threshold, but fall behind the new threshold and the classification manipulation yields negative payoffs. The only beneficiaries are the profitable firms that do not manipulate under the new accounting policy. This is because 1) there is less pooling, and 2) the excess supply lowers the cost of capital \( r \), thereby reducing the equilibrium repayment \( D(r) \) and thus the expected financing cost.

5. Discussion and Empirical Implications

5.1 The Role of Accounting Standard and Enforcement

In our model, the role played by an official threshold (as an accounting standard) \( \theta_s \) appears to be minimal. Instead, the cost of capital, which is determined by the demand for and supply of capital seems sufficient to elicit a report that fully complies with the standard. However, this argument is incomplete because the cost of capital can serve as an efficient device in the market only when the appropriate accounting standards and enforcement are in place; i.e., the cost of capital and accounting standard/enforcement are complementary means of achieving efficient aggregate investment in an economy.

Our simple general equilibrium framework renders it nontrivial to evaluate policies such as lenient or stringent accounting due to externalities among investors and firms. Upon the implementation of a new policy, both the demand for and supply of capital change, thereby generating different implications for welfare of the investor and of firms. As shown in Proposition 5, the reason why policy changes generate both positive and negative externalities is
because the financial reporting equilibrium (found in Proposition 2) is already Pareto efficient. Thus, there is no way to make every stakeholder better off by changing accounting policies (either $\theta_s$ or $\pi$). A similar point is also discussed in Bernanke and Gertler (1990) and Zhang (2013). While it is possible to increase the total social welfare by changing the policy variables, some groups of stakeholders benefit from the new policy, and other groups of stakeholders bear extra costs. Needless to say, our comparative static results imply that a general equilibrium interaction would be the first step in evaluating the net benefit to society, in order to progress towards better accounting policies that are applied to multiple, heterogeneous parties.

5.2 Empirical Predictions

Based on our results (Proposition 2, 3, 4 and 5), we provide the following testable predictions. First, the greater the aggregate wealth of an economy is, 1) the lower the cost of capital is, 2) the more classification manipulation occurs, and 3) the greater aggregate investment is. Second, the more conservative an accounting standard is or the stronger regulatory enforcement is, 1) the lower the cost of capital is, 2) the less classification manipulation occurs, and 3) the smaller aggregate investment is, thus 4) the more the investor consumes and the less she invests. Turning to the change in accounting policies, in particular either as the accounting standard becomes more stringent (or more conservative) or as the enforcement level becomes stronger, 1) the cost of capital decreases, 2) the volume of classification manipulations declines, and 3) accordingly, aggregate investment decreases. Moreover, upon this change, 4) the investor’s consumption increases, but her savings decreases.

There are numerous empirical studies testing cross-country differences in the cost of capital, which has been shown to vary depending on legal environments (Francis, Khurana, and Pereira, 2005; Hail and Leuz, 2006), the role of accounting conservatism on macroeconomic performance and policy change (Crawley, 2015), or cross-country differences in the investor’s intertemporal substitutions (Havranek, Horvath, Irsova and Rusnak, 2015). Building on these studies, one could empirically investigate macroeconomic outcomes of accounting standards and regulatory enforcement that our model predicts. This general equilibrium approach would be a kickoff toward a better assessment of such accounting policies (Leuz and Wysocki, 2016).
6. Conclusion

In this paper, we propose a simple general equilibrium model of investment financing with information asymmetry in order to better assess accounting standards and regulatory enforcement in an economy. Such standards and enforcement are uniformly applied to multiple heterogeneous firms that demand capital by preparing accounting reports. Firms can potentially engage in classifications manipulation, which increases the market demand. The investor makes choices between consumption and savings based on disclosed accounting reports and the expected returns to savings. The equilibrium cost of capital, the volume of classifications manipulation, aggregate investment, and consumption and savings decisions are determined in the market depending on an accounting standard and enforcement level. We show that any changes in policies (relating to either standards or enforcement) generate externalities among different stakeholders, which suggests that the full assessment of accounting policy choices requires a general equilibrium approach.

References


Appendix A.

Proof of Lemma 1.

Given \( \omega \), consider the first best case in which the realized \( \theta \) is fully observable. Then, for firm \( \theta \), the investors can charge different repayment \( D(\theta) \) subject to:

\[
\theta \times D(\theta) = (1 + r)I, \Rightarrow D(\theta) = \frac{1 + r}{2\theta}X,
\]

where, we use \( I = \frac{X}{2} \). Given this, the cost of capital \( r \) is determined by the market clearing condition. There are two cases. If \( \omega \) is sufficiently high, then for social efficiency, all projects with non-negative NPV must be undertaken. i.e., taking into account the cost of project implementation \( b \geq 0 \), for all firms \( \theta \) that make the non-negative NPV, the capital is provided.

\[
\theta(X - D(\theta)) \geq b.
\]

For this to be the case, the savings capital must be sufficiently high that:

\[
(1 - \theta^{FB})I \leq \omega - \frac{1}{1 + r} \Rightarrow (1 - \theta^{FB})I + \frac{1}{1 + r} \leq \omega.
\]

The marginal firm \( \theta^{FB} \) must earn zero profit in equilibrium. If \( \omega \) is sufficiently high that investor is willing to save even at \( r = 0 \), then

\[
\theta^{FB}(X - D(\theta^{FB})) = b \iff \theta^{FB} = \frac{1}{2} \left( \frac{2b}{X} + 1 \right).
\]

Notice that if the participation cost \( b = 0 \), then \( \theta^{FB} = \frac{1}{2} \). Thus, \( r^{FB} = 0 \) and \( \theta^{FB} = \frac{1}{2} \left( \frac{2b}{X} + 1 \right) \) are an equilibrium outcome if

\[
\omega \geq \left(1 - \frac{1}{2} \left( \frac{2b}{X} + 1 \right) \right) I + 1 = \frac{X - b}{2} + 1 \equiv \overline{\omega}.
\]

On the other hand, if \( \omega < \overline{\omega} \), then \( r^{FB} > 0 \) is required to induce investor’s savings. As before, the marginal firm \( \theta^{FB} \) must earn zero profit taking into account the cost of capital:

\[
\theta^{FB} \left( X - \frac{1 + r}{2\theta^{FB}}X \right) = b \iff \theta^{FB} = \frac{1}{2} \left( \frac{2b}{X} + 1 + r^{FB} \right).
\]
$r^{FB} > 0$ is determined by the market clearing condition:

$$
\left(1 - \frac{1}{2} \frac{2b}{X} + 1 + r^{FB}\right) l = \omega - \frac{1}{1 + r^{FB}} \frac{1}{a}
$$

$$
\Leftrightarrow \frac{X(1 - r^{FB})}{4} - \frac{b}{2} + 1 = \omega - \frac{1}{1 + r^{FB}} \frac{1}{a}.
$$

Since the left hand side is decreasing in $r^{FB}$, whereas the right hand side is increasing in $r^{FB}$, there exists a unique $r^{FB}$ that satisfies the equality.

Q.E.D.

**Proof of Lemma 2.**

We first derive the maximum willingness to pay of firm $\theta$ to characterize aggregate demand. Then, we show that this maximum willingness to pay is concave increasing in $\theta$. Let $\tilde{\theta}_T(r)$ denote the marginal firm the investor anticipates. For a given $r$ (since firms are price takers, they will take $r$ as given), the repayment $D$ must satisfy:

$$
E[\theta | \theta \geq \tilde{\theta}_T(r)]D = (1 + r)l \Rightarrow D = \frac{1 + r}{1 + \tilde{\theta}_T(r)} X,
$$

where we use $E[\theta]X = l, l = \frac{X}{2}$. To find the demand function, consider the payoff of firm $\theta \geq \theta_s$ at the repayment $D$.

$$
\theta(X - D) - b = \theta \left( X - \frac{1 + r}{1 + \tilde{\theta}_T(r)} X \right) - b = \frac{\theta X}{1 + \tilde{\theta}_T(r)} (\tilde{\theta}_T(r) - r) - b.
$$

Similarly, consider the payoff of firm $\theta < \theta_s$ at the repayment $D$.

$$
\theta(X - D) - b - \pi(\theta_s - \theta) = \theta \left( X - \frac{1 + r}{1 + \tilde{\theta}_T(r)} X \right) - b - \pi(\theta_s - \theta)
$$

$$
= \frac{\theta X}{1 + \tilde{\theta}_T(r)} (\tilde{\theta}_T(r) - r) - b - \pi(\theta_s - \theta).
$$

(6)

(7)
In equilibrium, $\theta_T(r) = \theta_T(r)$. Given $r$, the marginal firm $\theta_T(r)$ is found by equalizing the payoff (6) or (7) to zero. Equivalently, the zero profit condition is written as a quadratic equation. First consider the case in which the marginal firm does not manipulate:

$$X \theta_T(r)^2 - (b + rX)\theta_T(r) - b = 0 \Rightarrow$$

$$\theta_T(r) = \frac{b + rX + \sqrt{(b + rX)^2 + 4bX}}{2X} = v(r) + \sqrt{v(r)^2 + \frac{b}{2X}}$$

where $v(r) = \frac{b + rX}{2X}$ represents the success probability of the non-manipulating firm that obtains the minimum payoff from implementing a project. Firm $\theta = v(r)$ obtains negative payoff from implementing a project at the cost of capital $r$, and the marginal firm $\theta = \theta_T(r)$ earns zero payoff.

Next, consider the case in which the marginal firm does manipulate:

$$(X + \pi)\theta_T(r)^2 - (b + rX - \pi(1 - \theta_s))\theta_T(r) - (b + \pi\theta_s) = 0 \Rightarrow$$

$$\theta_T(r) = \frac{b + rX - \pi(1 - \theta_s) + \sqrt{(b + rX - \pi(1 - \theta_s))^2 + 4(b + \pi\theta_s)(X + \pi)}}{2(X + \pi)}$$

$$= v_T(r) + \sqrt{v_T(r)^2 + \frac{b + \pi\theta_s}{2(X + \pi)}},$$

where $v_T(r) = \frac{b + rX - \pi(1 - \theta_s)}{2(X + \pi)}$ represents the success probability of the manipulating firm that obtains the minimum payoff from implementing a project.

Q.E.D.

**Proof of Maximum investment when there is no accounting standard**

If there is no way to report, the aggregate investment obtains the maximum since there is no means of preventing firms from receiving funding. We use superscript N to denote the no accounting standard case. The marginal firm $\theta^N$ is determined by:
\[ \theta^N(X - D^N) - b = \frac{\theta^N}{1 + \theta^N} X (\theta^N - r^N) - b = 0 \]

\[ \Rightarrow \theta^N = \frac{r^N X + b + \sqrt{(r^N X + b)^2 + 4 b X}}{2X}. \]

Here, the expression for the marginal firm is the same as Lemma 2 except there is no classifications manipulation cost. The reason why there is self-screening is because of the opportunity cost of \( b \). The cost of capital \( r \) is the market clearing rate which depends on the endowment \( \omega \). The aggregate investment \((1 - \theta^N)I\) is maximum because the only obstacle is the opportunity cost. While the aggregate investment obtains its maximum, it does not necessarily maximize the aggregate welfare due to inefficient pooling (cross-subsidization). Since the aggregate investment obtains its maximum, \( r^N \) also obtains its maximum value.

Q.E.D.

**Proof of Proposition 3.**

Let \( \theta_T \) denote the success probability of the marginal firm. Then \( D = \frac{1 + r}{1 + \theta_T} X \). Thus, for firm \( \theta \geq \theta_T \), the expected repayment is:

\[ \theta \times D = \theta \frac{1 + r}{1 + \theta_T} X, \]

which increases in \( \theta \).

To show that there exists a case in which firms with negative NPV projects obtain capital by manipulation, it is sufficient to compare \( \theta^{FB} \) with \( \theta_T(r) \) for a given \( \omega \). Conditional on that there are some negative NPV projects are funded, the demand is greater than the first best. Thus, the market clearing cost of capital is greater than the first best. Let \( r^{FB} \), \( r \) denote the cost of capital under \( \theta^{FB} \) and \( \theta_T(r) \), respectively.

Observe that if \( \theta_s < \theta^{FB} \), then some negative NPV firms can obtain report \( B \). These firms do not have to manipulate to have access to capital. In this case, it is possible that the marginal firm engages in manipulation, however, given that the standard already includes the negative NPV
firms, it is sufficient to show that there exists the marginal firm that does not manipulate but with success probability less than $\theta_{FB}$ is able to obtain capital. That is:

$$\theta_{FB} > \theta_T(r) \iff \frac{1 + r_{FB}}{2} + \frac{b}{X} > \frac{r}{2} + \frac{b}{2X} + \sqrt{\frac{(b + rX)^2}{4X^2} + \frac{b}{X}}$$

$$\iff r < \frac{(1 + r_{FB})^2 - 2b}{b} \frac{X}{X} + (1 + r_{FB}) \equiv r.$$ 

For $r$ to be well-defined, the below condition is required.

$$(1 + r_{FB})^2 - 2b > 0 \iff \frac{b}{X} < \frac{(1 + r_{FB})^2}{4}.$$ 

Therefore, provided that the standard is less than the marginal firm’s success probability under the first best and that $\frac{b}{X}$ is sufficiently small, when $r < \underline{r}$, some negative NPV projects are funded. The case $r < \underline{r}$ happens when the endowment is sufficiently high.

On the other hand, if $\theta_s \geq \theta_{FB}$, then some positive NPV firms have to manipulate in order to report $B$. Thus, we need to compare $\theta_{FB}$ with the marginal firm $\theta_T(r)$ that manipulates its report. i.e.,:

$$\theta_{FB} > \theta_T(r)$$

$$\iff \theta_{FB} > \frac{b + rX - \pi(1 - \theta_s)}{2(X + \pi)} + \sqrt{\frac{(b + rX - \pi(1 - \theta_s))^2}{4(X + \pi)^2} + \frac{b + \pi\theta_s}{X + \pi}}$$

$$\iff \theta_{FB} - \frac{b + rX - \pi(1 - \theta_s)}{2(X + \pi)} > \sqrt{\frac{(b + rX - \pi(1 - \theta_s))^2}{4(X + \pi)^2} + \frac{b + \pi\theta_s}{X + \pi}}$$

$$\iff r < \frac{1}{X} \left( \frac{(X + \pi)\theta_{FB}^2}{\theta_{FB}} - (1 + \theta_{FB})(b + \pi\theta_s) + \pi \right) \equiv \underline{r}'.$$

For $\underline{r}'$ to be well-defined, the below condition is required.
\[
(X + \pi)\theta^{FB} - \frac{(1 + \theta^{FB})(b + \pi \theta_s)}{\theta^{FB}} + \pi > 0
\]
\[
\Leftrightarrow \frac{b}{X} < \frac{\theta^{FB^2}}{1 + \theta^{FB}} - \frac{\pi}{X} (\theta_s - \theta^{FB}).
\]

i.e., Provided that the standard is greater than or equal to the marginal firm’s success probability under the first best and \(\frac{b}{X}\) is sufficiently small, when \(r < r'\), some firms that manipulate their reports and have lower success probability have access to capital. Again, the case \(r < r'\) happens when the endowment is sufficiently high.

Q.E.D.

Proof of Proposition 1.

To show the existence and uniqueness, it is convenient to use inverse demand and supply curves. We use the continuity and monotonicity of inverse demand and supply curves. We derive conditions with respect to the endowment \(\omega\) that guarantee the two curves crossing for market clearing. Let \(Q^* = (1 - \theta_T(r^*))I\) denote the aggregate investment.

Case 1) \(Q^* \leq (1 - \theta_s)I\)

In this case, the equilibrium investment is determined by the demand by non-manipulating firms since the supply of capital is not enough. In the main text, we derived the payoff of the marginal firm. To see how the inverse demand (i.e., cost of capital \(r\)) changes as the marginal firm changes, we use \(\theta_T\) to denote the marginal firm, \(Q = (1 - \theta_T)I\) as the aggregate demand as an independent variable and \(r(Q)\) as a function of \(Q\):

\[
\theta_T\left(X - \frac{1 + r(Q)}{1 + \theta_T}X\right) - b = \frac{\theta_TX}{1 + \theta_T} (\theta_T - r(Q)) - b = 0,
\]
\[
\Rightarrow r(Q) = \theta_T - \frac{b(1 + \theta_T)}{\theta_TX} = 1 - \frac{Q}{I} - \frac{b}{(1 - \frac{Q}{I})X}.
\]

The last equality uses \(\theta_T = 1 - \frac{Q}{I}\).
Notice that $r(Q)$ is decreasing in $Q$:

$$\frac{dr(Q)}{dQ} = -\frac{1}{I} - \frac{bl}{(I-Q)^2X} < 0.$$  

Thus, the maximum cost of capital is found at $Q = 0$ (or, $\theta_T = 1$), which is $r(0) = 1 - \frac{2b}{X}$, and the maximum cost of capital decreases monotonically as $Q$ increases. It is worth noting that conceptually, as cost of capital increases, the total demand for capital decreases. However, since we denote the inverse demand with respect to the aggregate demand, we have that the cost of capital decreases as the demand increases. When $Q = (1 - \theta_s)l$ (i.e., $\theta_T = \theta_s$), the cost of capital is $r(Q) = \theta_s - \frac{b(1+\theta_s)}{\theta_sX}$.

On the other hand, from the investor’s optimal consumption and savings choices, we know

$$s(r) = \omega - \frac{1}{1+} = Q.$$  

Rearrange the savings choice with respect to the aggregate supply, $Q$, and denote $r(Q)$ as a function of aggregate supply, we have the following inverse supply curve:

$$r(Q) = \frac{1}{(\omega - Q)^a} - 1.$$  

Conceptually, as interest rate increases, savings increases as well. From the inverse supply curve, since we denote the interest rate as a function of savings, we have $\frac{dr(Q)}{dQ} = a \left(\frac{1}{\omega - Q}\right)^{1+a} > 0$, the interest rate increases as the supply increases.

For the two curves to cross at $Q^* \leq (1 - \theta_s)l$, the inverse supply curve at $Q = (1 - \theta_s)l$ must be greater than or equal to the inverse demand curve at $Q = (1 - \theta_s)l$:

$$(\omega - Q)^{-a} - 1\big|_{Q=(1-\theta_s)l} \geq \theta_s - \frac{b(1 + \theta_s)}{\theta_sX}$$  

$$\Leftrightarrow \omega \leq (1 - \theta_s)l + \left(1 + \theta_s - \frac{b(1 + \theta_s)}{\theta_sX}\right)^{-\frac{1}{a}} \equiv \omega.$$  

36
Thus, if \( \omega \leq \omega \), then the two curves cross at \( Q^* \leq (1 - \theta_s)I \). Since \( Q^* = (1 - \theta_T)I \), the success probability of the equilibrium marginal firm \( \theta_T \) is greater than or equal to \( \theta_s \): \( \theta_T \geq \theta_s \).

To see whether \( r^* > 0 \), notice that the marginal firm \( \theta_T \) makes zero profit in equilibrium, thus,

\[
 r^* = \theta_T - \frac{b(1 + \theta_T)}{\theta_TX}.
\]

Recall we assume that \( \frac{b}{X} = \frac{\theta_o^2}{1 + \theta_o} \) where \( \theta_o < \theta_s \) and observe that \( \frac{\theta^2}{1 + \theta} \) is increasing in \( \theta \). Thus, we conclude that:

\[
 r^* > 0 \iff \frac{\theta_o^2}{1 + \theta_o} > \frac{b}{X}, \text{ which is true since } \theta_T \geq \theta_s > \theta_o.
\]

As we derived in Lemma 2, \( \theta_T(r^*) = v(r^*) + \sqrt{v(r^*)^2 + \frac{b}{X}} \). Every firm \( \theta \geq \theta_T(r^*) \) obtains capital at \( r^* \). Since \( \theta_T(r^*) > \theta_s \), there is no classifications manipulation.

Case 2) \( Q^* > (1 - \theta_s)I \)

In this case, the equilibrium investment includes the demand by manipulating firms. The inverse supply curve we derived in Case 1 remains the same. However, the firms that manipulate their classifications must incur the cost of manipulating, thereby having different maximum willingness to pay for capital. Thus, we need to derive the inverse demand curve for the manipulating firms.

\[
 \theta_T\left(X - \frac{1 + r(Q)}{1 + \theta_T}X\right) - b - \pi(\theta_s - \theta_T) = \frac{\theta_TX}{1 + \theta_T} (\theta_T - r(Q)) - b - \pi(\theta_s - \theta_T) = 0,
\]

\[
 \Rightarrow r(Q) = \theta_T - \frac{(b + \pi(\theta_s - \theta_T))(1 + \theta_T)}{\theta_TX}
\]

\[
 = 1 - \frac{Q}{I} - \frac{\left(b + \pi\left(\theta_s - 1 + \frac{Q}{I}\right)\right)(2 - \frac{Q}{I})}{(1 - \frac{Q}{I})X}.
\]

Combined with the result in Case 1, the inverse demand curve is characterized as:
\[ r(Q) = \begin{cases} 
1 - \frac{Q}{I} - \frac{b(2 - \frac{Q}{T})}{(1 - \frac{Q}{T})X} & \text{if } Q \leq (1 - \theta_s)I \\
1 - \frac{Q}{I} - \frac{b + \pi(\theta - 1 + \frac{Q}{T})}{(1 - \frac{Q}{T})X} & \text{if } Q > (1 - \theta_s)I.
\end{cases} \]

i.e., there is a kink at \( Q = (1 - \theta_s)I \). We showed that when \( Q \leq (1 - \theta_s)I \), \( r(Q) \) is decreasing in \( Q \) in Case 1. When \( Q > (1 - \theta_s)I \):

\[
\frac{dr(Q)}{dQ} = \frac{1}{I} - \frac{\pi}{IX} - \frac{(b + \pi\theta_s)}{(I - Q)^2X} < 0.
\]

Thus, \( r(Q) \) is decreasing in \( Q \) after the kink at \( Q = (1 - \theta_s)I \). Compared to non-manipulating firm’s cost of capital, the manipulating firm’s cost of capital is more sensitive to \( Q \). Again, conceptually, the demand for capital from manipulating firms decrease faster than non-manipulating firms due to manipulating cost. But, since we use the inverse demand, we compare \( \frac{dr(Q)}{dQ} \).

There are two cases to consider: the inverse demand curve and supply curve crosses after the kink at \( Q^* = (1 - \theta_T)I > (1 - \theta_s)I \), or they never cross at \( Q^* = (1 - \theta_T)I \).

First, for crossing to exist, the inverse supply at the maximum demand level \( Q = (1 - \theta_T)I \) when \( r = 0 \) must be greater than \( r = 0 \): capital is provided to some manipulating firms but not every manipulating firm.

\[
(\omega - Q)^{-\alpha} - 1|_{Q=(1-\theta_T)I,r=0} > 0 \Rightarrow \omega < (1 - \theta_T)I + 1|_{r=0} \equiv \bar{\omega}.
\]

From Lemma 2, \( \theta_T \) at \( r = 0 \) is:

\[
\theta_T = v_T(0) + \sqrt{v_T(0)^2 + \frac{b + \pi\theta_s}{2(X + \pi)}},
\]

where \( v_T(0) = \frac{b - \pi(1 - \theta_s)}{2(X + \pi)} \). Thus,
\[
\overline{\omega} = \left(1 - v_T(0) - \sqrt{v_T(0)^2 + \frac{b + \pi \theta_s}{2(X + \pi)}}\right)l + 1.
\]

Second, for crossing not to exist, the inverse supply at the maximum demand level \( Q = (1 - \theta_T)l \) when \( r = 0 \) must less than or equal to \( r = 0 \): capital is provided to all manipulating firms:

\[
(\omega - Q)^{-a} - 1|_{Q=(1-\theta_T)l,r=0} \leq 0 \quad \iff \quad \omega \geq \overline{\omega}.
\]

All together, if \( \underline{\omega} < \omega < \overline{\omega} \), then crossing happens at \( Q^* = (1 - \theta_T)l > (1 - \theta_s)l \). To show \( r^* > 0 \), assume by contradiction that at \( r^* = 0 \), the market clears when \( \underline{\omega} < \omega < \overline{\omega} \). Due to assumption that \( \theta_T(r = 0) \) is the last firm that is willing to manipulate its classification when the cost of capital is zero, the marginal firm \( \theta_T(r^*) > \theta_T(0) \) makes strictly positive payoff. Then, firm \( \theta_T(r^*) - \epsilon, \epsilon > 0 \) will deviate by demanding capital and still generating positive payoff. Since the investor’s utility is an increasing function of the cost of capital, this deviation makes the investor strictly better off. This deviation continues until the marginal firm makes zero profit, and those newly joined firms (i.e., deviating firms \( \theta_T(r^*) - \epsilon \)) are also better off because they have access to funding. This is contradiction that the market clears at \( r^* = 0 \). Therefore, \( r^* > 0 \) when \( \underline{\omega} < \omega < \overline{\omega} \). When \( \omega \geq \overline{\omega} \). Then, in equilibrium, the marginal firm is \( \theta_T(0) \) and \( r^* = 0 \).

The firms \( \theta < \theta_T(0) \) cannot deviate as it gives them strictly negative payoff.

As we derived in Lemma 2, \( \theta_T(r^*) = v_T(r^*) + \sqrt{v_T(r^*)^2 + \frac{b + \pi \theta_s}{X + \pi}} \). Every firm \( \theta \geq \theta_T(r^*) \) obtains capital at \( r^* \). Since \( \theta_T(r^*) < \theta_s \), firms \( \theta \in [\theta_T(r^*), \theta_s) \) manipulate their classifications.

Since we show that crossing is uniquely defined whenever \( r^* > 0 \), \( r^* \) is uniquely determined by the market clearing condition,

\[
(1 - \theta_T(r^*))l = \omega - \frac{\frac{1}{a}}{1 + r^*}.
\]

The investor’s consumption and savings are fully characterized by \( r^* \).

Q.E.D.
Proof of Proposition 4.

We use the implicit function theorem to conduct comparative statics analysis.

Let $Q^S(\omega, a, r)$, $Q^D(\theta_s, \pi, X, r)$ denote the supply and demand function, respectively. We use the superscript to denote supply and demand and subscript to denote the partial derivative. In our model:

\[
Q^S_\omega = \frac{\partial}{\partial \omega} Q^S(\omega, a, r) > 0, \quad Q^S_a = \frac{\partial}{\partial a} Q^S(\omega, a, r) < 0, \quad Q^S_r = \frac{\partial}{\partial r} Q^S(\omega, a, r) > 0,
\]

\[
Q^D_{\theta_s} = \frac{\partial Q^D(\theta_s, \pi, X, r)}{\partial \theta_s} < 0, \quad Q^D_\pi = \frac{\partial Q^D(\theta_s, \pi, X, r)}{\partial \pi} < 0,
\]

\[
Q^D_X = \frac{\partial Q^D(\theta_s, \pi, X, r)}{\partial X} > 0, \quad Q^D_r = \frac{\partial Q^D(\theta_s, \pi, X, r)}{\partial r} < 0.
\]

The reason why $Q^D_{\theta_s} < 0$ is because for $Q^D(\theta_s, \pi, X, r) \leq (1 - \theta_s)I$, the demand strictly decreases as $\theta_s$ increases. For $Q^D(\theta_s, \pi, X, r) = (1 - \theta_T)I > (1 - \theta_s)I$, ceteris paribus, $\theta_T$ increases as $\theta_s$ increase, thus decreasing $Q^D$. Provided that $\pi$ plays a non-trivial role (i.e., enough endowment), the reason why $Q^D_\pi < 0$ is shown in the following claim, which is essentially in spirit similar with the proof of Proposition 1 in Holmstrom and Tirole (1997).

Claim. $Q^D_\pi < 0$

Proof of Claim) Assume by contradiction that $Q^D(\theta_s, \pi, X, r)$ increases as $\pi$ increases. The increase in $Q^D(\theta_s, \pi, X, r)$ means that the success probability of the marginal firm decreases. In the meantime, this increase in demand must be funded by the increase in capital, implying an increase in the interest rate. But, then, in order for the marginal firm to bear the increase in the cost of capital and the increase in the classification manipulation cost, the success probability of that marginal firm cannot decrease, i.e., the demand cannot increase. Contradiction. □

At an equilibrium state, we have

\[
Q^S(\omega, a, r) - Q^D(\theta_s, \pi, X, r) = 0,
\]

which determines $r$ as a function of $\omega, a, \theta_s, \pi$ and $X$ in equilibrium. For notational convenience, we drop the arguments and use the convention that the subscript denotes the partial derivative.
Knowing that \( r \) is a function of these exogenous variables, take the partial derivatives with respect to each exogenous variable for equation (5):

\[
\frac{\partial r}{\partial \theta_s} - \frac{\partial r}{\partial \pi} - \frac{\partial r}{\partial \omega} - \frac{\partial r}{\partial a} - \frac{\partial r}{\partial X} = 0,
\]

\[
\frac{\partial r}{\partial \theta_s} - \frac{\partial r}{\partial \pi} - \frac{\partial r}{\partial \omega} - \frac{\partial r}{\partial a} - \frac{\partial r}{\partial X} = 0.
\]

Rearrange these, then we have:

\[
(Q_r^s - Q_r^p) \frac{\partial r}{\partial \theta_s} = Q_{\theta_s}^p,
\]

\[
(Q_r^s - Q_r^p) \frac{\partial r}{\partial \pi} = Q_{\pi}^p,
\]

\[
(Q_r^s - Q_r^p) \frac{\partial r}{\partial \omega} = -Q_{\omega}^s,
\]

\[
(Q_r^s - Q_r^p) \frac{\partial r}{\partial a} = -Q_{a}^s,
\]

\[
(Q_r^s - Q_r^p) \frac{\partial r}{\partial X} = Q_{X}^p.
\]

Knowing that \( Q_r^s - Q_r^p > 0 \), we conclude:

\[
\frac{\partial r}{\partial \theta_s} = \frac{Q_{\theta_s}^p}{Q_r^s - Q_r^p} < 0, \quad \frac{\partial r}{\partial \pi} = \frac{Q_{\pi}^p}{Q_r^s - Q_r^p} \leq 0,
\]

\[
\frac{\partial r}{\partial \omega} = -\frac{Q_{\omega}^s}{Q_r^s - Q_r^p} < 0, \quad \frac{\partial r}{\partial a} = -\frac{Q_{a}^s}{Q_r^s - Q_r^p} > 0, \quad \frac{\partial r}{\partial X} = \frac{Q_{X}^p}{Q_r^s - Q_r^p} > 0.
\]
Therefore, the cost of capital decreases as the standard becomes stronger, the enforcement level becomes stronger, the endowment is greater, the motive for income smoothing becomes greater, and/or the production size becomes lower.

Q.E.D.

Proof of Proposition 4.

Since $\theta_T$ is a function of $r$ which is a function of $\theta_s$, take the total derivative of $\theta_T$ with respect to $\theta_s$:

$$
\frac{d\theta_T}{d\theta_s} = \frac{\partial \theta_T}{\partial \theta_s} + \frac{\partial \theta_T}{\partial r} \frac{dr}{d\theta_s} > 0 + > 0 < 0
$$

Similarly, take the total derivative of $\theta_T$ with respect to $\pi$:

$$
\frac{d\theta_T}{d\pi} = \frac{\partial \theta_T}{\partial \pi} + \frac{\partial \theta_T}{\partial r} \frac{dr}{d\pi} > 0 + > 0 \leq 0
$$

In this case, whether the increase in $\theta_s$ or $\pi$ appears to be indeterminate. We directly show that indeed the total effect with respect to each $\theta_s$ and $\pi$ on $\theta_T$ is positive. Assume by contradiction that the increase in $\theta_s$ lowers $\theta_T$. Thus, more firms manipulate upon the increase in the standard, which increases the demand. This demand must be funded by the increase in supply, which implies an increase in the cost of capital. But, then by the same reason for $D\theta_s < 0$, due to the increase in the cost of capital and increase in the manipulation cost (due to increase in $\theta_s$), the success probability of the marginal firm cannot go down. Thus, $\frac{d\theta_T}{d\theta_s} > 0$. Similarly, suppose that the increase in $\pi$ lowers $\theta_T$. By the the same reason, the increased demand must be funded by the increase in supply, which implies an increase in the cost of capital. But, then by the same reason for $D\pi \leq 0$, due to the increase in the cost of capital and increase in the manipulation cost, the success probability of the marginal firm cannot go down. Therefore, $\frac{d\theta_T}{d\pi} > 0$.

Similarly, take the total derivative of $\theta_T$ with respect to $\omega$:

$$
\frac{d\theta_T}{d\omega} = \frac{\partial \theta_T}{\partial \omega} + \frac{\partial \theta_T}{\partial r} \frac{dr}{d\omega} < 0.
$$
In this case, it is easy to see that as \( \omega \) increases, more firms engage in classification manipulation.

While \( \theta_T \) is not a function of \( a \), the parameter \( a \) affects the cost of capital \( r \), thus:

\[
\frac{d\theta_T}{da} = \frac{\partial \theta_T}{\partial a} + \frac{\partial \theta_T}{\partial r} \frac{dr}{da} > 0.
\]

Similarly, take the total derivative of \( \theta_T \) with respect to \( X \):

\[
\frac{d\theta_T}{dX} = \frac{\partial \theta_T}{\partial X} + \frac{\partial \theta_T}{\partial r} \frac{dr}{dX} > 0.
\]

As before, assume by contradiction that the increase in \( X \) boosts \( \theta_T \). Thus, less firms manipulate upon the increase in \( X \), which decreases the demand. This decrease in demand lowers the cost of capital. But, then the increase in the marginal benefit of implementing project with the reduction in cost of capital leads to a strictly positive payoff of the marginal firm, which is contradiction. Thus, \( \frac{d\theta_T}{dX} < 0 \).

Q.E.D.

Proof of Proposition 5.

Let \( V(\theta, r), U(c(r), s(r)) \), respectively, denote the utility of firm \( \theta \) and the utility of the investor at the cost of capital \( r \). Since \( \underline{\omega} < \omega < \bar{\omega} \), in equilibrium \( \theta_T(r^*) < \theta_s \) and some manipulating firms have access to funding.

1) Lenient accounting: lower \( \theta_s \) or \( \pi \)

i) We first consider the lower official threshold for accounting standard. To distinguish the new standard from the old one, we use \( \theta_s' \). Since \( \frac{\partial \theta_T}{\partial \theta_s} > 0 \), the changed standard \( \theta_s' \) creates another Nash standard \( \theta_T(r^{**}) < \theta_T(r^*) \) with another cost of capital \( r^{**} > r^* \). The reason why \( \theta_T(r^{**}) < \theta_T(r^*) \) is because the firms \( \theta \in [\theta_T(r^*) , \theta_s) \) that previously manipulate their classifications incur less manipulation cost. Thus, the marginal firm under the new standard \( \theta_s' \) lowers the success probability of the marginal firm \( \theta_T(r^{**}) \) that earns zero profit. Moreover, the
reason why \( r^{**} > r^* \) is because for the increased demand to be financed, the new equilibrium must elicit more savings by increasing the interest rate.

Since \( r^{**} > r^* \), the investor’s savings increases: \( \omega - \frac{1}{1 + r^{**}}^{1/a} \). Moreover, the investor’s payoff strictly increases because their utility is a strictly increasing function of \( r \):

\[
U(c(r), s(r)) = \frac{1}{1 - a} \left( \frac{1}{1 + r} \right)^{\frac{1-a}{a}} + (1 + r) \left( \omega - \frac{1}{1 + r} \right)^{\frac{1}{a}},
\]

\[
\frac{dU}{dr} = \omega - \left( \frac{1}{1 + r} \right)^{\frac{1}{a}} > 0.
\]

Thus, the new standard (and the new shadow standard) increases the investor’s welfare.

To analyze the welfare of firms, recall that firm \( \theta \)'s payoff for a given Nash standard \( \theta_T(r) \) and the cost of capital \( r \):

\[
V(\theta, r) = \begin{cases} 
\frac{\theta X}{1 + \theta_T} (\theta_T - r) - b & \text{for } \theta \geq \theta_s, \\
\frac{\theta X}{1 + \theta_T} (\theta_T - r) - b - \pi(\theta_s - \theta) & \text{for } \theta \in [\theta_T(r^*), \theta_s), \\
0 & \text{for } \theta < \theta_T.
\end{cases}
\]

It is immediate to see that as \( \theta \geq \theta_T \) increases, firm \( \theta \)'s utility increases: \( \frac{\partial V(\theta, r)}{\partial \theta} > 0 \). We first consider the payoff of firm \( \theta \in [\theta_T(r^*), \theta'_s) \) under the new standard \( \theta'_s \):

\[
V(\theta, r) = \frac{\theta X}{1 + \theta_T(r^{**})} (\theta_T(r^{**}) - r^{**}) - b - \pi(\theta'_s - \theta), \quad \theta \in [\theta_T(r^*), \theta'_s).
\]

As discussed, \( \theta_T(r^{**}) \) is less than \( \theta_T(r^*) \), while \( r^{**} \) increases, thus their payoff appears to be indeterminate under the new standard. However, firm \( \theta_T(r^*) \) enjoys strictly positive payoff since the new marginal firm \( \theta_T(r^{**}) \) earns zero payoff and the funded firm’s payoff is strictly increasing in \( \theta \). Then, due to monotonicity of \( V(\theta, r) \) with respect to \( \theta \), the payoff of firm \( \theta > \theta_T(r^*) \) is strictly increasing under the new standard as well. Thus, the new standard benefits the firms that manipulate their classifications under the old standard \( \theta_s \).
By the same reason $\frac{\partial V(\theta, r)}{\partial \theta} > 0$, every firm $\theta \in [\theta_s', \theta_s)$ enjoys the increase in payoff since they no longer incur the manipulation costs (although the cost of capital increases).

Now, consider firm $\theta \in [\theta_T(r^{**}), \theta_T(r^*)]$. Clearly, the new standard makes these firms strictly better off as they now have access to funding with strictly positive payoff for all $\theta > \theta_T(r^{**})$. Finally, consider the payoff of firm $\theta \geq \theta_s$. Unlike the firms $\theta < \theta_s$, these profitable firms’ payoffs decrease since the increase in demand boosts the cost of capital $r^{**}$ but decrease in the new Nash standard $\theta_T(r^{**})$ boosts the repayment. Thus, previously non-manipulating firms are strictly worse off under the loosened standard.

ii) We now consider the lower $\pi$. The proof is the same as the lenient standard. Let $\pi'$ denote the new enforcement level. Since $\frac{\partial \theta_T}{\partial \pi} > 0$, the changed enforcement creates another shadow standard $\theta_T(r^{**})$ with $r^{**} > r^*$ due to the increase in demand. Due to this increase in savings return, the investor consumes less, whereas increasing her savings. Since $\frac{\partial U}{\partial r} > 0$, the investor is better off.

Since the new marginal firm earns zero profit, the firms with success probability $\theta \in [\theta_T(r^*), \theta_s)$ enjoys the increase in payoff due to the decrease in manipulation cost (although the cost of capital increases, the net effect is positive). The firms with success probability $\theta \in [\theta_T(r^{**}), \theta_T(r^*)]$ are better off since they have access to funding. The only firms that suffer from this change is again the firms with success probability $\theta \geq \theta_s$ due to the increase in cost of capital and inefficient pooling (thus increase in repayment).

2) Stringent accounting: greater $\theta_s$ or $\pi$

i) We first consider the increase in $\theta_s$. To distinguish the new standard, let $\theta_s'$ denote the new threshold. This shift makes excess supply. i.e., $D(\theta_s', \pi, X, r) < S(\omega, a, r)$. Thus, $S(\omega, a, r)$ would have to go down for the market clearing condition, implying a decrease in the interest rate $r^{**} < r^*$. Due to the decrease in the interest rate, the investor’s consumption increases: $\frac{1}{1 + r^{**}}$. As we showed in the lenient accounting case, since $\frac{\partial U}{\partial r} > 0$, the decrease in the interest rate obviously decreases the investor welfare. Thus, the investor is hurt by the strengthened standard.
Meanwhile, since $\frac{\partial \theta_T}{\partial T} > 0$, the new shadow standard increase: $\theta_T(r^{**}) > \theta_T(r^*)$. The firms’ payoffs are then:

$$V(\theta, r) = \begin{cases} 
\frac{\theta X}{1 + \theta_T} (\theta_T - r) - b & \text{for } \theta \geq \theta_s', \\
\frac{\theta X}{1 + \theta_T} (\theta_T - r) - b - \pi(\theta_s' - \theta) & \text{for } \theta \in [\max\{\theta_T(r^{**}), \theta_s\}, \theta_s'), \\
\frac{\theta X}{1 + \theta_T} (\theta_T - r) - b - \pi(\theta_s' - \theta) \text{ if } \max\{\theta_T(r^{**}), \theta_s\} = \theta_s, & \text{for } \theta \in [\min\{\theta_T(r^{**}), \theta_s\}, \max\{\theta_T(r^{**}), \theta_s\}), \\
0 & \text{for } \theta \in [\theta_T(r^*), \min\{\theta_T(r^{**}), \theta_s\}), \\
0 & \text{for } \theta < \theta_T(r^*).
\end{cases}$$

If $\max\{\theta_T(r^{**}), \theta_s\} = \theta_s$, then firms $\theta \in [\theta_T(r^{**}), \theta_s)$ can obtain capital by manipulating their reports. However, due to the increase in manipulation cost, they are worse off under the new standard since the firm $\theta_T(r^{**})$ earns zero profit under the new standard, whereas it used to earn strictly positive payoff under the old standard. Due to this shift in payoffs at the bottom, firms $\theta \in [\theta_s, \theta_s')$ are also worse off since they have to incur manipulation costs in order to obtain capital whereas it used to obtain capital without manipulation under the old standard.

If $\max\{\theta_T(r^{**}), \theta_s\} = \theta_T(r^*)$, then firms $\theta \in [\theta_s, \theta_T(r^{**})]$ are worse off since they no longer have access to capital. By the same reason, firms $\theta \in [\theta_T(r^*), \theta_s)$ are worse off since they cannot obtain capital. For firms with success probability $\theta \geq \theta_s'$, they are better off since

$$\frac{\partial V(\theta, r)}{\partial \theta_T} = \frac{(1+r^{**})\theta X}{(1+\theta_T)^2} > 0.$$ 

ii) We now consider the greater $\pi$. The shift in $\pi$ increases the success probability of the marginal firm $\theta_T(r^{**}) > \theta_T(r^*)$. The proof is again the same as the stringent standard. Using the same notation before, due to excess supply, we have $r^{**} < r^*$. Thus, the investor consumes more and saves less. Since $\frac{dt}{dr} > 0$, the decrease in the interest rate obviously decreases the investor welfare. Thus, the investor is hurt by the strengthened enforcement.

Turning to the firms’ welfare, as in the stringent standard, we have the similar payoffs depending on $\theta$. If $\pi$ increases sufficiently that it eliminates the manipulation, i.e., $\theta_T(r^{**}) = \theta_s$, then the implications for firms’ payoffs are simple: the firms $\theta \geq \theta_s$ are strictly better off due to the
decrease in cost of capital and mitigated pooling, whereas firms $\theta \in [\theta_T(r^*), \theta_s)$ are worse off since they have no long access to capital. If the shift in $\pi$ still keeps some manipulating firms as the demand for capital, the payoffs are:

$$V(\theta, r) = \begin{cases} 
\frac{\theta X}{1 + \theta_T} (\theta_T - r) - b & \text{for } \theta \geq \theta_s, \\
\frac{\theta X}{1 + \theta_T} (\theta_T - r) - b - \pi (\theta_s - \theta) & \text{for } \theta \in [\theta_T(r^{**}), \theta_s), \\
0 & \text{for } \theta \in [\theta_T(r^*), \theta_T(r^{**})], \\
0 & \text{for } \theta < \theta_T(r^*). 
\end{cases}$$

The firms with $\theta \geq \theta_s$ are the only beneficiary since they can enjoy the reduced cost of capital and reduced repayment. The firms with $\theta \in [\theta_T(r^{**}), \theta_s)$ suffer from this change due to the increase in manipulation cost: firm $\theta_T(r^{**})$ used to earn a strictly positive payoff under the old enforcement, and now earns zero profit under the new standard. The firms with $\theta \in [\theta_T(r^*), \theta_T(r^{**})]$ are hurt as well since they no longer have access to capital.

Q.E.D

Appendix B.

Since we depicted the change in the cost of capital with respect to $\omega$ in Figure 2, here we depict the numerical examples of comparative statics with respect to $\theta_s, \pi, a, X$. The upward slopping curve is the inverse supply curve. The downward slopping curves are the inverse demand curves where the dashed line is the inverse demand curve when an exogenous parameter changes.

1. $\frac{\partial y^*}{\partial \theta_s} > 0$: The figure below depicts the change in cost of capital when the threshold $\theta_s$ increases for the following parameter values: change in $\theta_s = 0.5$ to $\theta_s = 0.6$, and $l = 1.5, b = 0.1, \pi = 0.3, a = 0.5$. 

47
2. $\frac{\partial \gamma^*}{\partial \pi} \geq 0$: The figure below depicts the change in cost of capital when the enforcement $\pi$ increases for the following parameter values: change in $\pi = 0.3$ to $\pi = 1$, and $l = 1.5, b = 0.1, \theta_s = 0.5, a = 0.5$.

3. $\frac{\partial \gamma^*}{\partial a} > 0$: The figure below depicts the change in cost of capital when the investor’s elasticity of substitution decreases (increase in $a$) for the following parameter values: change in $a = 0.3$ to $\pi = 0.7$, and $l = 1.5, b = 0.1, \theta_s = 0.5, \pi = 0.3$. 
4. $\frac{\partial y^*}{\partial x} > 0$: The figure below depicts the change in cost of capital when the project outcome increases for the following parameter values: change in $X = 3$ to $X = 3.2$, and $b = 0.1, \theta_s = 0.5, \pi = 0.3, a = 0.5$. Since $X = 2I$ in our model, the change in $X$ also corresponds to the change in capital $I$. 

\[ y \] 

\[ Q \]