The Economic Consequences of Discrete Recognition and Continuous Measurement

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Abstract

Discrete recognition is a long-standing and ubiquitous accounting practice, but it has been widely criticized for suppressing information and inducing accounting-motivated transactions. We study a model to examine the economic consequences of shifting away from discrete recognition to a continuous measurement approach. Without manipulation, discrete recognition is less informative than the continuous approach. However, the continuous regime induces more manipulation. The equilibrium informativeness is determined by both the accounting standard and endogenous manipulation. Discrete recognition is more informative than its continuous counterpart precisely when manipulation is a severe threat. We respond to the call in Kothari, Ramanna, and Skinner (2010) for using positive accounting theory to explain certain long-standing accounting practices. We also discuss the model’s implications for fair value accounting.

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1 Introduction

Recognition, defined as the process of admitting information into the basic financial statements (SFAC 5), is a fundamental accounting concept and long-standing accounting practice. It determines when and how a transaction is included in the financial statements. Depending on recognition criteria, an expenditure may be capitalized as an asset on the balance sheet or expensed on the income statement, the future payment under a lease contract may be recorded as a liability on the balance sheet or left off the balance sheet, and the delivery of products to a customer may lead to revenue recognition or no effect on the income statement at all.

Recognition entails discrete accounting consequences. Consider as an example a long-term asset lease contract in which a firm promises to pay annual installments for a period that accounts for $t$ percent of the useful life of the leased asset. Under the previous standard, the promised future payment is recognized as a lease liability if $t$ exceeds 75 but may remain off the balance sheet if $t$ stays just below 75. When $t$ changes from 74 to 76, the amount of lease liability on the balance sheet jumps disproportionately. In other words, the accounting consequence around $t = 75$ is discrete. In contrast, the new lease standard (ASC 842) takes a more continuous approach. The recognized lease liability is roughly proportional to $t$. The same change of $t$ around 75 leads only to a small change in the amount of the lease liability on the balance sheet. Thus, the accounting consequence around $t = 75$ becomes more continuous.

Dye, Glover, and Sunder (2014) (hereafter DGS) have argued that recognition suppresses information in two ways. First, it directly excludes information. In the lease example, the promised payment may remain off the balance sheet when $t$ stays just below 75. Second, the discreteness in accounting consequences motivates firms to engage in activities that are designed primarily for financial reporting benefits. We use the broad term evidence management to refer to such activities. Evidence management further degrades the report’s informativeness.\footnote{Evidence management has been empirically documented in the accounting literature, including, among others, Imhoff and Thomas (1988) (for leases), Lys and Vincent (1995) (for merger and acquisition), Engel, Erickson, and Maydew (1999) (for hybrid securities), and Dechow and Shakespear (2009) (for securitization). It has also long been a major concern for standard setters and regulators. For example, in its report to the}
In light of these shortcomings, DGS recommend that standard setters “adopt a continuous approach” whereby a transaction’s expected economic value is recognized without truncation. Consistent with this recommendation, recent accounting standards have been moving towards a more continuous approach, as exemplified in the new lease accounting standard. The deliberation of contingent liabilities is also moving significantly towards this continuous approach.

How do we evaluate this shift towards the continuous approach to standard setting? The positive accounting theory (e.g., Watts and Zimmerman (1986, 1990) and Kothari, Ramanna, and Skinner (2010)) cautions us not to abandon long-standing accounting practice without careful economy analysis. In light of the long history and prevalence of discrete recognition in accounting practice, we would like to understand both the discrete and continuous regimes. How do we evaluate the consequences of the continuous approach to standard setting? Is the shift towards a continuous approach more consistent with the stated goal of financial reporting of providing more useful information to investors? How will it affect firms’ evidence management? What are the efficiency consequences?

We present a model to compare the informativeness and efficiency of a recognition regime and a continuous regime. In the model, the manager engages in evidence management to influence the mapping from the state (a transaction’s economics substance) to evidence (a transaction’s characteristics), and the accounting system converts evidence to an accounting report. Evidence management reduces the correlation between the state and its accounting evidence, while the reporting process determines how much accounting evidence is communicated in the report. In particular, relative to the continuous regime, recognition excludes some evidence (information) from the report. After receiving the report, the investor makes an investment decision. We define efficiency as a weighted average of the expected payoffs to the manager and to the investor.

Our main result is that discrete recognition generates more information and higher efficiency than continuous measurement if and only if evidence management poses a severe threat. If evidence management were absent, then the continuous regime would always be

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Congress on the financial reporting of off-balance sheet arrangements, the first recommendation the Securities and Exchange Commission (the SEC) makes is “to discourage transactions and transaction structures primarily motivated by accounting and reporting concerns, rather than economics” (see SEC (2005)).
more informative than the discrete recognition regime. If and only if the manager’s evidence management cost is sufficiently low, the discrete recognition regime is more efficient than the continuous regime. The intuition is as follows. On one hand, for a given evidence distribution, the recognition regime communicates less information than the continuous regime, as suggested by conventional wisdom. On the other hand, the recognition regime induces less evidence management than the continuous regime. The manager engages in evidence management to influence the report, because the investor’s decision is sensitive to the report. All else equal, since the discrete recognition regime reveals less evidence to the investor, it also mitigates the manager’s incentive for evidence management. In other words, the intensity of evidence management is increasing in the informativeness of the accounting system. Overall, the report’s equilibrium informativeness is jointly determined by the manager’s evidence management and by the accounting regime. Therefore, there is a trade-off between the discrete recognition regime and the continuous regime, leading to our main result.

Empirically, our model predicts more evidence management when we move away from the recognition regime and more towards the continuous regime. Note that this prediction is not inconsistent with the compelling evidence on evidence management around recognition thresholds we have discussed in footnote 1. In the recognition regime, evidence management is intensified around the recognition threshold but muted away from the threshold. Its intensity around the threshold makes it more conspicuous and easier to document empirically. However, in the continuous regime, evidence management may arise not only for firms around the threshold but also in other ranges. While individually it might be less noticeable, it could be even more damaging to the report’s informativeness in aggregation. Such empirical bias demands careful research design to test our prediction.

Our primary contribution is to provide a rationale for the long-standing accounting practice of discrete recognition. DGS have developed a formal framework to demonstrate that discrete recognition degrades the report’s informativeness. Based on this result, their major recommendation is to abandon discrete recognition and “adopt a continuous approach.” Apart from their main argument, they have also pointed out that they might be missing the merits of discrete recognition and made some conjectures. Our paper proposes one merit of discrete recognition and provides a more balanced evaluation of the shift to the continuous approach.
Contrary to the main conclusion in DGS, we show that the continuous approach generates more informative financial reporting if and only if the threat of evidence management is not severe.\(^2\)

Glover (2013) has pointed out another merit of discrete recognition. It facilitates auditing and improves verifiability. Under the recognition approach auditors can focus their attention on the most verifiable features of transactions (around the thresholds). A shift towards the continuous approach can reduce verifiability and increase information asymmetries about verifiability, which can have qualitative impact on the nature of the contracting problem between investors and managers (e.g., Glover, Ijiri, Levine, and Liang (2005)).

Our second contribution is to extend the two-step representation of accounting measure-ment (e.g., Dye (2002) and Gao (2013a)). Motivated by the prevalence of discrete recognition in accounting, Dye (2002) has pioneered a tractable model to study its consequences. While this framework has been adopted by a number of papers to study various issues in standard setting, the literature so far has taken the discrete recognition as given so as to focus on its consequences.\(^3\) By specifying the conditions for the optimality of discrete recognition, we lend support to the prior literature that takes the discrete recognition as a starting point. In Section 5 we elaborate the innovations in our model that have made it possible to identify the conditions under which discrete recognition is efficient.

Third, our paper also makes a methodological contribution by using integral precision as a criterion of ranking information systems. This powerful criterion was recently developed in Gamuza and Penalva (2010) and first used in accounting by Marinovic (2013). Evidence management does not reduce the report’s informativeness in the Blackwell sense, but it does so in the sense of integral precision. Moreover, it is both necessary and sufficient that a more integral precise information system leads to higher decision efficiency. Thus, as far as the

\(^2\)Since our model endogenizes accounting evidence that accounting standards classify, our approach re-
sponds to Demski’s calls in his presidential address to American Accounting Association (Demski (2004)) for providing micro-foundations of equilibrium expectations in evaluating accounting policies. He argues that “the FASB has [...] a penchant for focusing on a type of transaction and then determining the proper accounting treatment of that transaction. It does not [...] overly concern itself with the supply of transactions if it pro-
scribes one particular accounting treatment. [...] The FASB’s Conceptual Framework strikes me as [...] being built upon a foundation that sidesteps micro foundations of the underlying choices, and largely inadequate for scholarly purpose.”

decision-making usefulness is concerned, integral precision is a strong enough informativeness criterion. It adds a useful tool to our toolbox to deal with informativeness issues.

Finally, our paper is also related to the agency theory. The broad economic force in our paper, that controlling a manager’s ex ante incentive to engage in evidence management requires the inefficient use of information ex post, is a recurring theme in the agency literature. Recognition, by suppressing information ex-post, substitutes for the decision maker’s commitment to mute her reaction to accounting reports. As such, this paper could be viewed as an application of the agency theory to accounting standard design. For example, Arya, Glover, and Sivaramakrishnan (1997) explore a similar trade-off in studying the optimal design of information system. In a model with both control and decision problems, they show that when the principal cannot commit to the use of information ex-post, the optimal information system suppresses some information. The suppression of ex-post information serves as a substitute for the principal’s lack of commitment. For another example, Chen, Hemmer, and Zhang (2007) study the role of conservatism in dampening the manager’s incentive to manipulate earnings. They argue that conservatism mutes the market’s reaction to bad news, reduces the manager’s benefit from earnings management, and, hence, results in less earnings management. Yet another example is Mittendorf (2010) who studies the role of audit thresholds in the misreporting of private information. The commitment to tolerating misreporting within the materiality threshold makes the threat of punishing egregious misreporting above the threshold more effective.

The rest of the paper proceeds as follows. Section 2 describes the model. Section 3 solves the model and characterizes the equilibrium under each regime. Section 4 compares the efficiency of the two regimes and presents our main result. Section 5 explains our model’s relation to the prior literature, Section 6 discusses our model’s implications and Section 7 concludes.

2 The model

We augment the evidence management model à la Dye (2002) with an ex-ante manipulation technology. In the baseline mode, there are two players, a manager and an investor. For ease
of reference, we refer to the manager as “he” and the investor as “she.” When we refer to a random variable, we use “~" to denote the random variable itself and the same notation without “~" to denote its realization. There is also a passive player, labeled as the accountant, who administers the measurement process but does not play any strategic role. Each moves once in the following sequences.

1. At date 0, an accounting system is installed to report transactions. We specify the accounting systems in more detail below.

2. At date 1, the manager engages in unobservable evidence management \( m \) that influences accounting evidence \( t \).

3. At date 2, state \( \omega \) is realized but not observable to anyone. The accountant collects accounting evidence \( t \) and converts it to report \( s \) according to the prevailing accounting system.

4. At date 3, the investor observes report \( s \) and makes a decision \( I \). \( I \) and \( \omega \) jointly determine the payoffs of the investor and the manager, \( v(\omega, I) \) and \( u(\omega, I) \), respectively.

The firm’s viability, denoted as \( \tilde{\omega} \), is either high \( (H) \) with probability \( q_H \) or low \( (L) \) with probability \( q_L \equiv 1 - q_H \), i.e., \( \omega \in \{H, L\} \). We normalize \( H = 1 \) and \( L = 0 \) for simplicity. Viability \( \tilde{\omega} \) is not observable but measured by an accounting system. Thus, we also call \( \tilde{\omega} \) the economic substance of a representative transaction or simply the state.  

The investor makes an investment decision \( I \). The future net cash flow to the investor is

\[
v = \omega I - \frac{\lambda}{2} I^2. \tag{1}\]

She benefits from an investment decision better matched to the firm’s true viability, as reflected in the interaction term \( \omega I \). The investment costs \( \frac{1}{2} \lambda I^2 \), with \( \lambda \) being a parameter.

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4For a theory about the externality of accounting standards and disclosure regulation, see Dye (1990), Admati and Pfleiderer (2000) and Dye and Sridhar (2008). We also abstract away from the political economy issues in accounting standards setting and refer readers to, among other, Dye and Sunder (2001), Bertomeu and Magee (2011, 2015a,b), Bertomeu and Cheynel (2013), and Friedman and Heinle (2016).

5Note that binary states do not imply that the optimal accounting rule generates binary signals because the decision is continuous. As we show later, continuous signals dominate binary ones when there is no manipulation.
of the adjustment cost. In contrast, the manager’s payoff is

\[ u = \beta I - \kappa K(m). \]  

\(^{(2)}\)

\(\beta I\) is the manager’s gross payoff from investment, while \(\kappa K(m)\) is the private cost of evidence management that will be explained later. Since the investor prefers to match the investment to the firm’s viability while the manager prefers larger investment, their interest is not fully aligned. The misalignment is captured by parameter \(\beta > 0\).

State \(\tilde{\omega}\) manifests itself in accounting evidence \(\tilde{t}\), with density \(f^{\tilde{w}}(t)\) over \([\tilde{t}_H, \tilde{t}_L]\). \(f^{\tilde{w}}(t)\) satisfies MLRP, i.e., \(\frac{f^{\tilde{w}}(t)}{f^{\tilde{w}}(\tilde{t})}\) is increasing in \(t\). Thus, evidence \(t\) is informative about the state in the sense of Milgrom (1981).\(^{6}\)

Accounting evidence \(t\) is converted into a report, denote as \(\tilde{s}\), according to the prevailing accounting regime. We consider two accounting regimes. In a discrete recognition regime, accounting evidence \(t\) is partitioned into two sets by a threshold \(T\), resulting in two reports:

\[ s = \begin{cases} h & \text{if } t \geq T \\ l & \text{if } t < T \end{cases}. \]  

\(^{(3)}\)

Threshold \(T\) could be arbitrary in our model.\(^{7}\) We assume that it is in the common support of \(f(t|H)\) and \(f(t|L)\), i.e., \(T \in [\tilde{t}_H, \tilde{t}_L]\), as other cases are trivial.

This representation of accounting recognition is simplistic. In practice, recognition is followed by measurement and thus the report \(\tilde{s}\) is less discrete. We consider an alternative form of the recognition regime in Section 4.3.

We also consider a continuous regime in which the accounting evidence is directly communicated to the investor. That is, there is no recognition and the transaction is directly measured. Thus, the resulting report is

\[ s = t, \text{ for any } t. \]  

\(^{(4)}\)

\(^{6}\)Dye (2002) standardizes the accounting evidence by introducing the probability of the state \(\phi\), which can be recovered from the distribution of state \(\tilde{\omega}\) and accounting evidence \(\tilde{t}\) by the Bayes rule, that is, \(\phi \equiv \Pr(\omega = H|t) = \frac{q_H f^{\tilde{w}}}{q_H f^{\tilde{w}} + q_L f^{\tilde{w}}}\).

\(^{7}\)See Dye (2002), Laux and Stocken (forthcoming) and Gao (2015) for studies of designing optimal thresholds.
Our main research question is to compare the discrete recognition regime with the continuous measurement regime. We denote the regime using a subscript $i \in \{C, D\}$, where $C$ denotes the continuous measurement regime and $D$ denotes the discrete recognition regime. We omit the subscript $i$ whenever no confusion arises.

The key friction in the accounting measurement process is that the manager can take costly actions to influence the evidence distribution. Specifically, before observing $\omega$, the manager can choose manipulation intensity $m \in [0, 1]$ that changes the evidence distribution from $f^\omega(t)$ to

$$f^\omega_m(t) = mf^H(t) + (1 - m)f^\omega(t).$$

If $m = 0$, the evidence distribution is not affected by evidence management, i.e., $f^\omega_m(t) = f^\omega(t)$. If $m = 1$, then the manager always receives accounting evidence that is not distinguishable from a high state, i.e., $f^\omega_m(t) = f^H(t)$. If $m \in (0, 1)$, evidence management improves the evidence distribution in the sense of first-order stochastic dominance. This ex-ante evidence management technology enables us to compare the two regimes in a tractable manner. We postpone the discussions and justification of this assumption to Section 5 after we have explained how our model works.

Given an accounting regime $i$, evidence management affects the final report $\tilde{s}$. We denote its probability density function (when $\tilde{s}$ is continuous) or probability function (when $\tilde{s}$ is discrete) as $g^\omega_m(s)$ for realization $s$.

Evidence management $m$ costs the manager $\kappa K(m)$ privately. $K(m)$ has the standard properties of a cost function. It is increasing and convex with $K(0) = K'(0) = 0$. In addition, we assume that $\frac{K'(m)}{K''(m)}$ is weakly increasing in $m$ on $(0, 1)$, or, equivalently that $K'(m)$ is weakly log-concave and that $\lim_{m \to 0} \frac{K(m)}{K''(m)} = 0$. The conditions are satisfied for common convex functions used in the literature (see Bagnoli and Bergstrom (2005)), e.g. $K(m) = m^n$ for $n > 1$.

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8The technology implies that evidence management does not improve the evidence distribution in the good state, that is, there is no “good evidence management.” If evidence management improves the evidence distributions in both states, our main results are qualitatively the same provided that the improvement is larger in the bad state than in the good state.

9The accounting system measures a firm’s existing transactions and past events, while the report in our model is useful for making future investment decisions. What is implicit in our model is that the firm’s profitability (state) in the past and in the future are correlated. Therefore, the report from the accounting system that measures the firm’s past transactions is also useful for investors to make new investment. For example, investors use income statement to make new investment decisions. To economize on notation, we have assumed that they are perfectly correlated.
\( n \geq 2, \kappa > 0 \) is a cost parameter that will be used for comparative statics later.

The solution concept is Perfect Bayesian Equilibrium (PBE). A PBE consists of the manager’s evidence management \( m^* \) and the investor’s investment strategy \( I^*(s) \) such that, for any given regime,

1. the manager chooses \( m^* \) to maximize \( U = E[u(m)] \);
2. the investor chooses \( I^*(s) \) to maximize \( V = E[v(I)|s] \);
3. The investor has rational expectations about the manager’s evidence management.

### 3 The equilibrium

For a given regime, we solve the model by backward induction. We first solve for the investor’s investment response given her conjecture about the manager’s manipulation. We then solve for the manager’s manipulation decision and finally impose the rational expectations requirement.

We first analyze the investor’s decision at date 3 after she receives report \( s \). Since she does not observe the manager’s manipulation, she conjectures that the manager has chosen \( \hat{m} \). Based on this conjecture, she chooses investment \( I \) to maximize her expected payoff \( E_\omega[v(I)|s; \hat{m}] \). Denote \( I^{BR} \) as her best investment response to report \( s \). The first-order condition results in

\[
I^{BR}(s; \hat{m}) = \arg \max E_\omega[v(I)|s; \hat{m}] = \frac{1}{\lambda} E[\omega|s; \hat{m}] .
\]  

(6)

The second-order condition is satisfied due to the concavity of the investment return function \( v(I) \). The investor’s best investment response characterized in equation (6) is intuitive. The investor invests more if the investment adjustment cost \( \lambda \) is lower or if her posterior belief about the state is higher.

At date 1 the manager anticipates the investor’s investment response \( I^{BR}(s; \hat{m}) \) at date 3 and chooses \( m \) to maximize his expected payoff \( E[u] \). With manipulation \( m \), the manager expects to receive evidence distribution \( f^{\omega'}_m(t) \) in equation (5) and its corresponding report distribution \( g^{\omega'}_m(s) \). The investor then responds to report \( s \) with investment \( I^{BR}(s; \hat{m}) \). Therefore, the manager’s expected payoff with manipulation \( m \) when the investor conjectures \( \hat{m} \)
is

\[ U(m; \tilde{m}) \equiv \beta E_\omega [ \int I^{BR}(s; \tilde{m}) g^\omega_m(s) ds ] - \kappa K(m). \quad (7) \]

Since the investor does not observe the manager’s actual choice of \( m \), her investment decision does not directly respond to \( m \). In other words, \( I^{BR}(s; \tilde{m}) \) is independent of \( m \). Instead, evidence management affects the manager’s expected payoff only through its effect on the report distribution \( g^\omega_m(s) \).

We now explain how \( g^\omega_m(s) \) is determined. In the continuous regime, \( s_C = t \) and thus report \( \tilde{s}_C \) has the same distribution as the evidence. This results in

\[ g^\omega_m(s_C) = f^\omega + m(f^H - f^\omega), \text{ for any } s_C. \quad (8) \]

Under the discrete recognition regime, report \( \tilde{s}_D \) has a binary distribution, \( i.e., s_D \in \{h, l\} \). We can write it out as:

\[ g^\omega_m(s_D) = \begin{cases} \int_{t < T} \{ f^\omega + m(f^H - f^\omega) \} dt & \text{if } s_D = l \\ \int_{t \geq T} \{ f^\omega + m(f^H - f^\omega) \} dt & \text{if } s_D = h \\ 0 & \text{otherwise} \end{cases}. \quad (9) \]

To highlight the effects of evidence management on the manager’s expected payoff, we define the manager’s expected payoff in state \( \omega \) when he does not manipulate (but the investor still conjectures \( \tilde{m} \)) as

\[ \Pi^\omega(\tilde{m}) \equiv U(0; \tilde{m}) = \beta \int I^{BR}(s; \tilde{m}) g^\omega(s) ds. \quad (10) \]

We can rewrite \( U(m; \tilde{m}) \) in equation (7) as

\[ U(m; \tilde{m}) = q_H \Pi^H(\tilde{m}) + q_L \Pi^L(\tilde{m}) + q_L m (\Pi^H(\tilde{m}) - \Pi^L(\tilde{m})) - \kappa K(m). \quad (11) \]

Equation (11) is intuitive. In the absence of evidence management, the manager’s base expected payoff is \( q_H \Pi^H(\tilde{m}) + q_L \Pi^L(\tilde{m}) \). Evidence management \( m \) improves the distribution of evidence from \( g^L(s) \) to \( g^H(s) \) with probability \( m \) in the low state, increasing the manager’s
expected payoff by $q_L \left( \Pi^H(\hat{m}) - \Pi^L(\hat{m}) \right)$ at the cost of $\kappa K(\hat{m})$.

Substituting the expressions of the distribution of report $\tilde{s}$, $g_m^\omega(s)$ from expression (8) and (9), into the manager’s objective function $U(m; \hat{m})$ in equation (11) and taking the first-order condition of $m$, we obtain the manager’s best evidence management response (to the investor’s conjecture of $\hat{m}$), $m^{BR}(\hat{m})$, as

$$q_L \left( \Pi^H(\hat{m}) - \Pi^L(\hat{m}) \right) - \kappa K'(m^{BR}(\hat{m})) = 0. \quad (12)$$

The second-order condition is satisfied because of the convexity of the cost function $K(m)$. Equation (12) explains the manager’s evidence management decision. Given the investor’s conjecture $\hat{m}$, the marginal benefit of manipulation is $q_L \left( \Pi^H(\hat{m}) - \Pi^L(\hat{m}) \right)$, the manager’s expected payoff difference with and without a favorable evidence distribution in the low state. The manager trades off this marginal benefit against the marginal cost of $\kappa K'$ to determine the best response $m^{BR}(\hat{m})$.

Finally, the investor has rational expectations about the manager’s evidence management. Even though she does not observe evidence management $m$, the investor’s conjecture is consistent with the manager’s actual choice in equilibrium. Thus, we obtain the equilibrium by solving the fixed point problem (equation (12)) of $m^{BR}(\hat{m}) = \hat{m}$. As we show in the proof, this equilibrium has a unique solution and we denote it as $m^*$.

Evaluating the investor’s investment best response in equation (6) at the equilibrium evidence management level $m^*$, we obtain the equilibrium investment function $I^*(s; m^*) = I^{BR}(s; \hat{m})|_{\hat{m}=m^*}$. The equilibrium $(I^*, m^*)$ is summarized below.

Proposition 1 The equilibrium consists of the manager’s evidence management $m^*$ and the investor’s investment decision $I^*(s)$. The equilibrium investment decision $I^*(s)$ is characterized by

$$I^*(s) = \frac{1}{\lambda} E[\omega|s; m^*]. \quad (13)$$

The interior equilibrium evidence management $m^*$ is characterized by the first-order condition

$$q_L \left( \Pi^H(m^*) - \Pi^L(m^*) \right) - \kappa K'(m^*) = 0. \quad (14)$$
$m^*$ increases in $\beta$ and decreases in $\kappa$ and $\lambda$.

We can now verify the impossibility theorem in DGS, that is, $m^* = 0$ cannot be an equilibrium. If the investor believes that the manager does not manipulate and thus takes the report at its face value, then the marginal benefit of manipulation is strictly positive. Since the marginal cost of manipulation is 0 at $m^* = 0$, the manager will deviate by engaging in some positive amount of evidence management.\textsuperscript{10}

The determinants of the interior $m^*$ are intuitive. $\beta$ measures the misalignment of interest between the manager and the investor. An increase in $\beta$ exacerbates the manager’s conflict of interest with the investor and thus induces the manager to engage in more evidence management. A higher $\kappa$ indicates a higher marginal cost of evidence management and thus leads to lower evidence management. Finally, $\lambda$ is the adjustment cost of the investment. It reduces the investor’s response to the report. As we can see from the equilibrium investment decision (equation (13)), $\lambda$ scales down the investment level $I^*$ for any given report $s$. Because the manager’s manipulation is driven by the investor’s response to the report, the equilibrium manipulation is decreasing in adjustment cost $\lambda$.

4 The comparison of the two accounting systems

Having solved the equilibria, we can compare them under two regimes. For a given regime, we have obtained a unique equilibrium characterized by $m^*$ and $I^*(s)$. We first examine the report’s equilibrium informational properties in the two regimes. These properties are the key building blocks to prove the subsequent results. We then present the main results about the efficiencies of the two regimes. We finish this section with an alternative characterization of the recognition regime.

\textsuperscript{10}All four conditions for the impossibility theorem in DGS are satisfied in our model. In particular, the manager has reasonable preference and the standards are typical because the equilibrium investment $I^*(s)$ is increasing in the report. The continuous evidence $t$ satisfies condition 3 and the continuous cost function with $K'(0) = 0$ meets condition 4. Thus, evidence management arises in equilibrium, that is, $m^* > 0$. 
4.1 The informational properties of the reports

The investor uses the reports in equilibrium because they are informative about the state \( \omega \). Report \( \tilde{s} \) is linked to state \( \omega \) through the accounting system, which can be characterized as follows:

\[
\text{state} (\omega) \quad \text{---} \quad \text{evidence} (\tilde{t}) \quad \text{---} \quad \text{report} (\tilde{s})
\]

\[
\text{evidence management} \quad \text{accounting standards}
\]

Report \( s \) is a function of evidence \( t \) whose distribution is determined by state \( \omega \) and evidence management \( m^* \). Thus, the report’s equilibrium informativeness with respect to the state is affected by both the manager’s evidence management and the accounting regime. Evidence management influences the mapping from state \( \omega \) to evidence \( t \), and the accounting regime shapes the mapping from evidence \( t \) to report \( s \).

To compare the informativeness of reports from each regime, we need criteria for informativeness. The classic criterion is Blackwell informativeness.

**Definition 1** \( X \) and \( Y \) are two signals about state \( \omega \) with respective p.d.f. (or p.f.) \( f_X(x|\omega) \) and \( f_Y(y|\omega) \). \( X \) is more Blackwell informative than \( Y \) if and only if there exists a conditional density \( n(y|x) \) such that

\[
f_Y(y|\omega) = \int_{\text{supp}(X)} n(y|x)f_X(x|\omega)dx.
\]

Definition 1 provides a formal expression to capture the underlying intuition that \( X \) is more Blackwell informative than \( Y \) if \( Y \) is a garbling of \( X \) with respect to the underlying state \( \omega \). Blackwell informativeness is a strong criterion and cannot be used to rank all information systems (e.g., Demski (1973)). In our model, an intuitive indicator of the report’s informativeness is evidence management. As \( m^* \) increases, the report becomes less informative under each regime. However, the report with lower equilibrium evidence management is not Blackwell more informative than its counterpart with higher equilibrium evidence management. Fortunately, the economics literature has developed an alternative criterion of informativeness, known as integral precision. It was developed by Ganuza and Penalva (2010) in the
context of ranking information systems the seller provides to buyers in an auction setting. Marinovic (2013) applies the notion to study accounting quality.

Definition 2 X and Y are two signals about state \( \omega \). \( X \) is more integral precise than \( Y \) if and only if \( E[\omega|Y] \) second-order stochastic dominates \( E[\omega|X] \).

Integral precision has an intuitive appeal. Information is useful because it moves the posterior belief away from the prior. The more informative a report is, the more dispersed the posterior belief distribution is. Second-order stochastic dominance is a natural measure of the dispersion of a distribution.

Integral precision is a weaker criterion than Blackwell informativeness and thus can rank more information systems. If \( X \) is more Blackwell informative than \( Y \), then \( X \) is also more integral precise than \( Y \), but the converse is not true.

Equipped with these definitions, we can study the equilibrium report’s properties. The equilibrium report \( s \) is a function of both the regime \( i \in \{C, D\} \) and the equilibrium evidence management \( m^* \).

Lemma 1 1. The investor’s average posterior belief about state \( \omega \) is equal to the prior:
\[
E[E[\omega|s_i;m^*]] = q_H, \text{ for any } m^* \text{ and } i.
\]
2. Given an evidence distribution, report \( \tilde{s}_C \) is Blackwell more informative than \( \tilde{s}_D \).
3. Given a report regime \( i \in \{C, D\} \), report \( \tilde{s}_i(m^*) \) is more integral precise as manipulation \( m^* \) decreases.

Part 1 of Lemma 1 is a reminder of the investor’s rational expectations requirement. Even though the investor does not observe manipulation \( m \), her conjecture is consistent with the manager’s actual choice in equilibrium. Conditional on the realizations of the reports, the investor’s posterior belief about the state will be updated to \( E[\omega|s_i;m^*] \). However, on average \( E[\omega|s_i;m^*] \) is expected to be equal to \( q_H \). In other words, the rational investor is not systematically misled by evidence management.

Part 2 of Lemma 1 confirms the conventional wisdom that recognition suppresses evidence. Given an evidence distribution, the report under the continuous regime reveals all
available evidence through \( s_C = t \). In contrast, the report under the recognition regime garbles the available evidence through the recognition rule characterized in scheme (3). Thus, the recognition regime suppresses evidence and is less informative even under the strongest criterion of Blackwell informativeness.

Part 3 of Lemma 1 is an intuitive but important result. It states that evidence management reduces the report’s informativeness (in the sense of integral precision) under both regimes. The purpose of financial reporting is to help the investor to differentiate the high state from the low, but evidence management makes it more likely that different states may generate the same reports. Since the investor rationally anticipates this contamination, she responds less to the report when the equilibrium manipulation is higher.

### 4.2 The main result

Now we are ready to present our main result. In each equilibrium, we can measure the ex ante payoffs to the investor and the manager. At date 0, the investor’s ex ante expected payoff is

\[
V \equiv E[v(I^*)] = \frac{\lambda}{2} E_s[(I^*)^2].
\]

\( V \) is also the investment efficiency as the investor relies on accounting reports to make investment decisions. The higher the investment efficiency, the higher the investor’s expected payoff.

The manager’s ex ante expected payoff is

\[
U \equiv U(m^*; m^*) = E[u(\omega, I^*)] = \beta E_s[I^*(s)] - \kappa K(m^*)
\]

\( U \) is increasing in the expected level of equilibrium investment \( I^* \) and decreasing in manipulation cost.

We define the efficiency as the weighted average of both the investor’s and the manager’s ex ante expected payoffs:

\[
W = \alpha V + (1 - \alpha) U, \text{ with } \alpha \in (0, 1).
\]
By allowing $\alpha = 1$, we include the extreme case that focuses only on the investor’s ex ante expected payoff. For reasons we will explain later, both the investor’s expected payoff and the efficiency go in the same direction as far as the comparison of the two regimes are concerned. Therefore, it is inconsequential which outcome variable we focus on. For completeness, we focus on the efficiency $W$.

Our main result is the following theorem.

**Theorem 1** There exists a unique $\bar{\kappa}$ such that the discrete recognition regime is more efficient than the continuous measurement regime (i.e., $W_D > W_C$) if and only if $\kappa < \bar{\kappa}$. $\bar{\kappa}$ decreases in $\lambda$ and $\alpha$ and increases in $\beta$.

Theorem 1 states that the efficiency could be higher in the recognition regime than in the continuous regime. Moreover, the recognition regime is more efficient if and only if the threat of evidence management is sufficiently high, i.e. $\kappa$ is sufficiently low. In other words, it is precisely the threat of evidence management that makes the recognition regime more efficient than the continuous regime.

For the rest of this subsection, we explain the proof of the theorem in four steps. First, given the same evidence distribution, the continuous regime is more efficient than the recognition regime. Second, evidence management reduces the efficiency in both regimes. Third, the equilibrium evidence management is higher in the continuous regime than in the recognition regime. As a result of these three steps, there is a trade-off between the two regimes. The recognition regime suppresses evidence but induces less evidence management. The last step is to show that this trade-off is well-behaved and generates the partition by $\bar{\kappa}$ presented in Theorem 1.

**Proposition 2** For the same evidence distribution, the efficiency is higher in the continuous regime than in the recognition regime. That is, for any $m^* \in [0, 1)$, $W_C(m^*) > W_D(m^*)$.

We first demonstrate that the manager’s expected payoff $U$ are the same in the two regimes when evidence management levels are equal. When the evidence management levels are the same across the two regimes, the evidence distributions are the same. The manager
benefits from the expected level of investment \( E_s[I^*(s)] \) and incurs the cost of evidence management \( \kappa K(m^*) \). When the evidence management levels are the same, the cost of evidence management is the same across the two regimes. Moreover, since investment \( I^* \) is linear in the investor’s conditional expectation of the state (equation (13) from Proposition 1), part 1 of Lemma 1 implies that the expected level of investment is the same across the two regimes as well. Since evidence management does not systematically mislead the investor’s belief about the state, it does not systematically mislead the investor’s investment decision either. Therefore, the manager’s expected payoff is the same across the two regimes when \( m^* \) is the same.

We then compare the investment efficiency \( V \) across the two regimes. The expression of \( V \) in equation (15) suggests that the investment efficiency is convex in the equilibrium investment level \( I^* \) and thus convex in the investor’s posterior belief \( E[\omega|\tilde{s}] \) (due to equation (13) from Proposition 1). This is intuitive. More efficient investment decision implies matching the investment decision more accurately to the state, that is, to invest more (less) in the high (low) state. Thus, the convexity simply means that the investment decision benefits from a more informative report (which induces a more volatile posterior belief distribution). By part 2 of Lemma 1, we know that, for the same evidence distribution, the continuous regime generates a more informative report and thus higher investment efficiency than the recognition regime.

Therefore, if evidence management is the same in the two regimes (which implies the same evidence distribution), the efficiency is higher in the continuous regime than in the recognition regime. Specifically, the manager’s expected payoffs are the same while the investment efficiency is higher in the continuous regime.

**Proposition 3** The efficiency is decreasing in evidence management in both regimes. That is, \( \frac{\partial W_i}{\partial m^i} < 0 \), for \( i \in \{C,D\} \).

We again start with the manager’s expected payoff \( U \). We have shown earlier that the expected level of investment is independent of evidence management. Moreover, the cost of evidence management is increasing in evidence management. Thus, the manager’s expected payoff is decreasing in evidence management. Even though the manager finds it optimal to
engage in evidence management, the investor rationally anticipates the manipulation and makes proper adjustment when interpreting the reports. As a result, the manager is stuck in this “bad” equilibrium, in which he engages in costly evidence management without systematically misleading the investor.

Now we turn to the investment efficiency. We have shown that $V$ is convex in the posterior belief $E[\omega|s]$ (combining equation (13) and equation (15)). Part 3 of Lemma 1 shows that evidence management reduces the report’s informativeness (in the sense of integral precision) in both regimes. Thus, evidence management reduces the investment efficiency.

Therefore, the efficiency in each regime is decreasing in evidence management. Specifically, both the manager’s and the investors’ expected payoffs are decreasing in evidence management.

**Proposition 4** The equilibrium evidence management is higher in the continuous regime than in the recognition regime. That is, $m_C^* > m_D^*$.

Proposition 4 is perhaps surprising at first glance, but it is a natural consequence of Proposition 2. Recall that the manager’s equilibrium evidence management is determined by equation (14). The manager engages in evidence management precisely because the investor is sensitive to the report, as captured by the term $\Pi^H(m^*) - \Pi^L(m^*)$. The manager manipulates more when the investor is more responsive to the report. For given evidence distribution, part 2 of Lemma 1 shows that the report from the continuous regime is more informative than that in the recognition regime, and Proposition 2 further shows that the investor’s investment decision is more responsive to the report from the continuous regime. Therefore, the manager’s incentive to manipulate is stronger as well. In other words, evidence management is more rampant in the continuous regime precisely because the continuous regime is more informative for a given evidence distribution.

Propositions 2, 3 and 4 together create a trade-off between the two regimes. Fixing the evidence distribution, the continuous regime generates a more informative report (Proposition 2). This induces the manager to manipulate more, which in turn reduces the report’s informativeness and the investor’s response to the report (Proposition 3). In equilibrium, the continuous regime induces more evidence management (Proposition 4).
Theorem 1 demonstrates that this trade-off generates an interior threshold \( \bar{\pi} \) that partitions the efficiency of the two regimes. To see the intuition, we could consider the two extremes. At one extreme when evidence management is absent, the continuous regime is more efficient, i.e., \( W_C(0) > W_D(0) \). At the other extreme, suppose \( m^*_C = 1 > m^*_D \) (which occurs with \( \kappa \) properly chosen). The report is completely uninformative under the continuous regime while still informative under the recognition regime. Then we have \( W_C(1) < W_D(m^*_D) \).

Therefore, there exist situations in which the continuous regime could be less efficient than the recognition regime. The last step to prove Theorem 1 is to show that the trade-off is well-behaved, that is, \( W_C(\kappa) = W_D(\kappa) \) has a unique solution \( \bar{\kappa} \). This part is technical and thus delegated to the appendix. We therefore have proved Theorem 1.

The determinants of the threshold \( \bar{\pi} \) in Theorem 1 further corroborate the main message that recognition is more efficient only when evidence management is a severe threat. A larger \( \bar{\pi} \) indicates that the recognition regime is more likely to dominate the continuous regime in efficiency. As we have discussed following Proposition 1, when the interest misalignment \( \beta \) is larger or when the adjustment cost \( \lambda \) is lower, the equilibrium evidence management is more severe. As a result, \( \bar{\pi} \) increases.

### 4.3 Recognition with measurement

In our baseline model we have compared the continuous regime with the recognition regime where the latter is modelled as a discrete classification. While discrete classifications have been used in the literature (e.g. Dye (2002), Gao (2013a), Laux and Stocken (forthcoming)), the accounting practice of recognition is often more complicated in reality. For example, in a typical revenue recognition transaction, if the revenue is not recognized, the firm reports 0. However, if the revenue is recognized, then the measurement process follows and the expected revenue is reported. Thus, the report is partially binary and partially continuous. We study this alternative characterization of the recognition regime. In particular, we modify equation (3) as follows

\[
s = \begin{cases} 
  t & \text{if } t \geq T \\
  l & \text{if } t < T.
\end{cases}
\]  

We show that our main result about the comparison of the two regimes are qualitatively
the same.

**Theorem 2** There exists a unique $\bar{\pi}_{alt}$ such that the recognition regime is more efficient than the continuous regime (i.e., $W_D > W_C$) if and only if $\bar{\pi} \prec \bar{\pi}_{alt}$. $\bar{\pi}_{alt}$ decreases in $\lambda$ and increases with $\beta$.

The reason Theorem 2 is qualitatively the same as Theorem 1 is because the driving force of our main result, that suppression of evidence in the recognition regime also mitigates the manager’s evidence management, still survives in this alternative characterization of recognition. Since the proofs are qualitatively the same, they are omitted here but available upon request.

5 The relation to the prior literature

Having explained the tractable trade-off between the discrete recognition and continuous measurement regimes, now we further discuss our model’s relation to that in Dye (2002) and DGS. Dye (2002) pioneered the evidence management model to study standard setting in the shadow of managerial opportunism. DGS apply the framework to analyze the effects of financial engineering on standard setting and demonstrate that financial engineering arises in equilibrium under a set of reasonable conditions. However, DGS are unable to evaluate the efficiency consequences of the discrete recognition regime, let alone comparing the continuous measurement and discrete recognition regimes.

A key difficulty that prevents them from doing so is the lack of a tractable proxy for the consequences of evidence management in their model, as they have discussed in detail. Conceptually, evidence management is inefficient for two reasons. Not only does it consume resources, it may also degrade the report’s informativeness and reduce the efficiency of decisions made on the basis of the report. In their models, evidence management is ex post: the manager chooses the amount of evidence management after observing the initial realizations of accounting evidence. To evaluate the standards ex ante, we have to aggregate the consequences of the ex post evidence management. This task is complicated and creates tractability issues. Specifically, there are two problems. First, the aggregation of ex post
evidence management in general is not technically tractable. Mathematically, the aggregate evidence management, \( \int m^*(t) dF(t) \), depends on the functional forms of both \( m^*(t) \) and \( F(t) \), and a tractable form is only available for special functions. As a result, Dye (2002) has confined the model to uniform distribution for \( F(t) \). Second, even if a tractable aggregator of ex post evidence management levels is available, it is not necessarily a proper measure of the informational consequences of evidence management. Specifically, evidence management induces accounting evidence distributions that are typically not Blackwell comparable. In other words, an increase in evidence management does not make the accounting evidence distribution less informative in Blackwell sense. In the absence of a proper aggregator for the consequences of evidence management, the comparison across the two regimes becomes intractable.

We overcome these two problems by modelling evidence management as an ex ante choice and by applying the informativeness criterion of integral precision. The manager chooses \( m \) without knowing \( \omega \) or initial evidence \( t \). This modeling choice negates the need for aggregating ex post evidence management since the manipulation choice is a scaler to start with. Moreover, the ex ante manipulation technology has an intuitive property that, ceteris paribus, the report’s informativeness is monotonically decreasing in the equilibrium level of evidence management (Lemma 1). Even though it is incorrect to judge an accounting standard by the amount of evidence management it induces (as explained in DGS), for a given standard, its efficiency is decreasing in the level of equilibrium evidence management. Finally, the ex-ante manipulation technology has an additional benefit of avoiding the tricky issues associated with ex post signaling.

Evidence management in practice can occur in all stages of a transaction, both before and after the transaction’s economic substance becomes clear to the manager. Thus, both ex ante and ex post evidence management are probably descriptive. Moreover, while it is technically convenient to assume that the manager chooses evidence management before receiving private information, what is essential for the ex ante evidence management modeling choice is that the manager does not have full control over the impact of evidence management on the final report. This feature implies that receivers cannot fully back out the impact of evidence management from only observing the report, leading to the result that evidence management
reduces the report’s informativeness. The manager’s residual uncertainty about the impact of his evidence management choices on the final report can result from various channels. For example, after evidence management, the auditor may challenge the manager’s accounting choices and force the manager to make adjustments to the final report. For another example, some evidence management choices are made before the transaction is finalized and thus their effects may interact with the realization of the transaction’s other characteristics, which may be outside the manager’s control. Bird, Karolyi, and Ruchti (2016) provide strong empirical evidence that managers face inherent uncertainty about how their accounting choices affect the final earnings.

Evidence management in our model is related to but differs from earnings management in the prior literature (e.g., Dye (1988), Arya, Glover, and Sunder (1998) and Arya and Glover (2008)). Consider a long-term asset lease contract in which the firm’s true economic lease liability is $\omega$ and the final report of lease liability on the balance sheet is $\sigma$. A typical ex-ante manipulation of the final report $\sigma$ takes the following form. It assumes a pre-manipulation relation $\sigma(\omega)$ and then allows the manager to change this relation to $\sigma(\omega; m)$. For example, $\sigma(\omega; m) = \omega + \varepsilon + m$. $\varepsilon$ is a noise term and $m$ is the manager’s ex-ante manipulation of the final report.

In contrast, the ex-ante manipulation of evidence requires the introduction of accounting evidence ($t$), a feature delivered by a two-step representation of accounting measurement: the first mapping from state $\omega$ to evidence $t$, and the second mapping from evidence $t$ to report $\sigma$. One example of accounting evidence in the lease contract above is the present value of future lease payment ($t$) computed for the purpose of implementing the accounting treatment (we call it “accounting present value” for short). If the prevailing standard utilizes the accounting present value, that is, $\sigma(t)$ varies with $t$, the manager can influence the accounting present value $t$ to indirectly affect the final report $\sigma$. Suppose an economically optimal payment structure is annual installments of equal amounts. Suppose the discount rate used for calculating the accounting present value is constant, but the (true) interest rate term structure at the end of the fiscal period could be upward or downward sloping with 80% and 20% chances, respectively. The ex-ante manipulation of evidence can take the following form. At the inception of the lease contract, the manager negotiates an ascending payment structure
at some extra cost $\kappa K(m)$. Relative to the benchmark structure (annual installments), the ascending structure leads to a lower accounting present value with 80% chance but a higher one with 20% chance. In the sense of first-order stochastic dominance, the ascending payment structure leads to a lower accounting present value $(t)$, which in turn results in a lower lease liability on the balance sheet $(s)$. In sum, evidence management provides a micro-foundation for the reduced-form manipulation of the final report. With this micro-foundation, it becomes possible to study how to design accounting standards in the shadow of managerial influence on the accounting process.

6 The empirical and policy implications

The model generates straightforward empirical implications. Accounting standards that adopt a more continuous approach always generate more evidence management. In addition, the standards generate less informative reports and less efficient investment decisions if and only if the transaction characteristics are sufficiently vulnerable to managerial manipulation. Moreover, the standards are more likely to result in these adverse effects if the manager’s manipulation incentive is higher or if the investment decisions are more sensitive to accounting reports (after controlling for the reports’ informativeness).

Our results provide several policy implications. First, our model formalizes one benefit of recognition (above and beyond measurement) and provides a counter argument to the tidal move towards a more continuous approach. This benefit is consistent with the prevalent use of recognition in extant accounting standards. We warn that a more continuous approach is desirable only for transactions that are robust to managerial manipulation. Second, our comparative statics about the determinants of the relative efficiency of the two regimes provide direct implications. We expect to see that recognition is more likely to prevail when the investment adjustment cost is lower or when the interest misalignment between the manager and the investor is more severe.

Finally, our results also have policy implications for the debate on fair value accounting.\textsuperscript{11}

\textsuperscript{11}We use the term “fair value” accounting to be consistent with the literature. However, the term is not appropriate for the debate, as explained in Sunder (2008). “Current value” accounting or “exit value” accounting might be more appropriate.
While the debate is multi-dimensional, one aspect of the fair value accounting resembles the continuous regime in our model. For example, a fair value approach to contingent liability moves us towards the continuous regime in our model. Proponents of fair value accounting often hold the view that fair value accounting, by providing more information, is more value relevant and helps investors to make better decisions (e.g., Barth (1994), Barth, Beaver, and Landsman (1996), and Barth, Hodder, and Stubben (2008). See Barth, Beaver, and Landsman (2001) for a review.) Opponents, however, argue that there is plenty of managerial discretion embedded in fair value estimates and managers will exploit it for their own benefits (e.g., Dietrich, Harris, and Muller (2000), Dechow, Myers, and Shakespeare (2010). See Kothari, Ramanna, and Skinner (2010) for a review). We show that the equilibrium informativeness of fair value accounting is conditional: it provides more information if and only if there is little room for managers to exploit the discretion.

7 The conclusion

The paper aims to explain a puzzle about recognition. Recognition before measurement is ubiquitous in accounting practice, even though it seems to degrade the report’s informativeness. Not only does recognition directly suppress information, it also induces evidence management that further degrades the report’s informativeness. We develop a model to compare a discrete recognition regime with a continuous measurement regime. We show that the discrete recognition regime is more efficient than the continuous regime if and only if the threat of evidence management is sufficiently severe. As such, we provide a rationale for the observed prevalence of recognition in accounting practice and offer a balanced view on the shift towards a more continuous approach in standard setting.

Our model introduces an ex-ante evidence management technology into the two-step representation of accounting measurement, whose implications for studying standard setting are elaborated in Gao (2013b). The ex-ante evidence management technology provides additional desirable features for modeling accounting standard setting. It overcomes the aggregation issue and provides a single measure of managerial influence on the accounting process. It delivers the intuitive property that manipulation decreases the report’s equilibrium informa-
tiveness. This technology could be exploited in other settings in future research.

Our model has a number of limitations that demand future research. First, we have modeled recognition in a restrictive manner. In practice, recognition is followed by measurement. Recognition determines whether or not to admit a transaction or event into the accounting system. Conditional on recognition, measurement determines the amounts of the elements by choosing a measurement attribute (such as historical cost, net realizable value and fair value (SFAC 5). Therefore, a transaction’s accounting consequences are jointly determined by recognition and measurement. In the recognition regime in our model, there could be multiple choices of measurement after recognition, among which we have considered a two-tiered measurement in the baseline model and a fair value measurement conditional on recognition in the extension in Section 4.3. In the continuous regime, there is no recognition and we have considered only fair value measurement. Future research may look at different combinations of recognition and measurement attributes.

Second, the model has been confined to the comparison of two regimes with and without recognition. It does not address the issue of the optimal design of the recognition regime. While binary recognition seems consistent with accounting practice, there is no obvious reason to believe that it is optimal among other possible forms. We leave the design of the optimal recognition regime to future research.

Third, we have assumed in the model that accounting reports are unambiguously valuable for the investor’s decisions (the demand side) and have instead focused on how the production of accounting reports is constrained by evidence management (the supply side). Recognition arises as an equilibrium response to evidence management. In contrast, there has been a large literature on biased performance measure and earnings management (see our literature review in Introduction). Most of this literature focuses on how the use of an accounting report is affected by earnings management. It would be desirable to better integrate the demand for and the supply of accounting information in the same model, which might generate new insights into the institutional features of accounting measurement.

Finally, discrete classifications are observed in many areas outside of accounting. Professors often give students discrete letter grades instead of continuous scores (e.g., Dubey and Geanakoplos (2010), Harbaugh and Rasmusen (2018)), and rating agencies issue discrete
ratings instead of continuous default probabilities (e.g., Goel and Thakor (2015)). Future research may compare and contrast these phenomenon with accounting recognition to better understand them.

8 Appendix

For convenience of exposition in the proof we will refer to $E[\omega|s;m]$ as $\gamma(s;m)$, i.e.

$$\gamma(s;m) \equiv E[\omega|s;m]$$

$$= \Pr(\omega = H|s;m)$$

$$= \frac{g^H(s)q_H}{g(s;m)}$$

$$= \frac{g^H(s)q_H}{(q_H + q_L m)g^H(s) + q_L (1-m)g^L(s)}$$

where we used Bayes’ rule in arriving at the equality.

Proof of Proposition 1:

**Proof.** We rewrite equation (13) and equation (14) here for convenience, i.e.

$$I^*(s;m^*) = \frac{1}{\lambda} \gamma(s;m^*)$$

and

$$q_L (\Pi^H(m^*) - \Pi^L(m^*)) - \kappa K'(m^*) = 0 \quad (18)$$

Denote $\Omega(m^*) \equiv q_L (\Pi^H(m^*) - \Pi^L(m^*)) - \kappa K'(m^*)$, i.e. the left hand side of equation (18), then its derivative with respect to $m^*$ results in

$$\frac{\partial \Omega(m^*)}{\partial m^*}$$

$$= q_L \frac{\beta}{\lambda} \int \gamma(s;m^*)[g^H(s) - g^L(s)]ds$$

$$\frac{\partial m^*}{\partial m^*}$$

$$= -q_L \frac{\beta}{\lambda} \int \left\{g^H(s)[g^H(s) - g^L(s)]^2 q_H q_L + g^L(s)q_L + [g^H(s) - g^L(s)]m^* q_L \right\} ds$$

$$-\kappa K''(m^*)$$

$$< 0$$

as both terms are negative.

Thus, the left hand side of equation (18) is decreasing in $m^*$.

When $m^* = 0$, $\Omega(0) = q_L (\Pi^H(0) - \Pi^L(0)) - \kappa K'(0) = q_L (\Pi^H(0) - \Pi^L(0)) > 0$.

Thus, when $m^*$ is interior, $m^*$ satisfies the first-order condition, i.e. equation (18). Differentiate both sides of equation (18) by $\beta$ results in

$$\frac{\partial \Omega}{\partial m^*} \frac{\partial m^*}{\partial \beta} + \frac{\partial \Omega}{\partial \beta} = 0$$

26
Thus,
\[ \frac{\partial m^*}{\partial \beta} = - \frac{\partial \Omega}{\partial m^*} \]

Note that we already show \( \frac{\partial \Omega}{\partial m^*} < 0 \). In addition,
\[ \frac{\partial \Omega}{\partial \beta} = \frac{1}{\lambda} \int \gamma(s; m^*)[g^H(s) - g^L(s)]ds > 0 \]
Thus
\[ \frac{\partial m^*}{\partial \beta} > 0 \]

Similarly, differentiating equation (18) by \( \kappa \) and \( \lambda \) results in \( m^* \) decreasing in \( \kappa \) and \( \lambda \). ■

Proof of Lemma 1:
Proof. We prove the three informational properties one by one.
First, we show that \( E[\gamma(s_i; m^*)] = q_H \), or, equivalently, \( E[\gamma(s_i; m^*)] = q_H \) for \( i = C, D \).
Note that this is essentially the law of iterated expectations. We provide the proof below for the sake of completeness.
Recall that in the continuous regime \( \gamma(s_C; m^*) = \frac{g^H(s_C)q_H}{g(s_C; m^*)} \) by Bayes’ Rule. Therefore
\[
E[\gamma(s_C, m^*)] = \int \gamma(s_C, m^*)g(s_C; m^*)ds_C = \gamma_hT(m^*)q_H
\]

Similarly, in the recognition regime, \( \gamma(l, m^*) = \frac{F^H(T)m^*}{F^H(T)}q_H \) and \( \gamma(h, m^*) = \frac{1-F^H(T)}{1-F^H(T)}q_H \) where \( F(t; m^*) \) is the unconditional cumulative distribution function of \( t \) given manipulation \( m^* \). Therefore
\[
E[\gamma(s_D, m^*)] = \gamma_hT(m^*)q_H + [1 - F^H(T)]q_H
\]

We next prove the second property, i.e. \( s_C \) Blackwell dominates \( s_D \).
Note that given any \( g(s_C|\omega) \), \( g(s_D = 1|\omega) = \int_{s_C \geq T} g(s_C|\omega)ds_C \) and \( g(s_D = 0|\omega) = \int_{s_C < T} g(s_C|\omega)ds_C \). Thus, define
\[
n(s_D = 1|s_C) = \begin{cases} 1 & \text{if } s_C \geq T \\ 0 & \text{if } s_C < T \end{cases}
\]
and
\[ n(s_D = 0|s_C) = \begin{cases} 
1 & \text{if } s_C < T \\
0 & \text{if } s_C \geq T 
\end{cases} \]

Then we have \( g(s_D|\omega) = \int n(s_D|s_C)g(\omega|s_C)ds_C \). Thus by Definition 1, \( s_C \) Blackwell dominates \( s_D \).

We finally prove the third property, i.e. \( s_i(m_i^*) \) is more integral precise than \( s_i(m_i^*) \) for \( i = C, D \). From Definition 2, we need to show that \( E[\omega|s_i(m_i^*)] \) second-order stochastically dominates (SOSD) \( E[\omega|s_i(m_i^*)] \), or, equivalently, the distribution of the random variable \( \tilde{g}(s_i;m_i^*) \) SOSD the distribution of \( \tilde{g}(s_i;m_i^*) \) for \( i = C, D \).

We first prove this for the continuous regime. To prove SOSD, we first need to write out the distribution of \( \tilde{g}(s_C,m^*) \). Note that since \( s_C = t \), \( g^\omega(s_C) = f^\omega(t) \) for \( \omega = H, L \).

\[ \gamma(s_C,m^*) = \Pr(\omega = H|s_C(m^*)) = \frac{g^H(s_C)}{g(s_C;m^*)q_H} = \frac{g^H(s_C)}{(q_H + q_Lm^*)f^H(t) + q_L(1-m^*)f^L(t)} = \frac{q_Hf^H(t)}{(q_H + q_Lm^*)f^H(t)} = \Psi\left(\frac{f^H(t)}{f^L(t)}; m^*\right) \]  

(19)

Since \( \frac{f^H(t)}{f^L(t)} \) is increasing in \( t \) by MLRP and \( \Psi\left(\frac{f^H(t)}{f^L(t)}; m^*\right) \) is increasing in \( \frac{f^H(t)}{f^L(t)} \), \( \Psi\left(\frac{f^H(t)}{f^L(t)}; m^*\right) \) is increasing in \( t \). This implies that the cumulative distribution function of \( \gamma(s_C,m^*) \), denoted as \( G^C(a;m^*) \), is

\[ G^C(a;m^*) = \Pr(\gamma(s_C,m^*) \leq a) = \Pr(t \leq t^*(a;m^*); m^*) = \int_{L}^{t^*(a;m^*)} f(t;m^*) dt = F(t^*(a;m^*); m^*) \]

where \( t^*(a;m^*) \) is the unique solution that satisfies \( \Psi\left(\frac{f^H(t^*(a;m^*))}{f^L(t^*(a;m^*))}; m^*\right) = a \) and \( f(t;m^*) \) is the unconditional probability density function of \( t \) given manipulation \( m^* \).

To show that \( \tilde{g}(s_C,m_1^*) \) SOSD \( \tilde{g}(s_C,m_2^*) \), we need to compare the integral \( \int_0^b G^C(x;m_1^*) dx \) and \( \int_0^b G^C(x;m_2^*) dx \) for arbitrary \( b > 0 \). It will be sufficient if we show that \( A^C(m^*) = \int_0^b G^C(x;m^*) dx \) is decreasing in \( m^* \) for arbitrary \( b > 0 \) and strictly decreasing on a set of non-zero measure. Denote \( \tilde{t} \) as the unique solution that satisfies \( f^H(\tilde{t}) = f^L(\tilde{t}) \) where the uniqueness is due to MLRP. MLRP also implies that \( \frac{f^H(t)}{f^L(t)} < 1 \) when \( t < \tilde{t} \) and \( \frac{f^H(t)}{f^L(t)} > 1 \) when \( t > \tilde{t} \). This implies that \( \gamma(s_C,m^*) \) is increasing in \( m^* \) when \( t < \tilde{t} \) and decreasing in \( m^* \) when \( t > \tilde{t} \). Figure ?? plots the cumulative distribution of \( \tilde{g} \) when
manipulation is high versus when manipulation is low. A conjecture from the graph is that \( \int_0^b G^C(x; m_1^*)dx \leq \int_0^b G^C(x; m_2^*)dx \), i.e. \( \tilde{\gamma}(s_C, m_1^*) \) SOSD \( \tilde{\gamma}(s_C, m_2^*) \), which we now show formally.

Before proceeding, we establish two algebraic facts that will be useful in the subsequent discussion.

First, \( t^*(a; m^*) \) is decreasing in \( m^* \) when \( a < q_H \), constant of \( m^* \) when \( a = q_H \) and increasing in \( m^* \) when \( a > q_H \). To see this, differentiate \( \Psi(\int_{f_H(t^*(a; m^*))}^{f_L(t^*(a; m^*))} dx) \) with respect to \( m^* \) results in

\[
\frac{\partial t^*(a; m^*)}{\partial m^*} = -\frac{\partial \Psi}{\partial \bar{y}}
\]

Thus, when \( t < \tilde{t} \), \( \frac{\partial \Psi}{\partial \bar{y}} > 0 \) and \( \frac{\partial t^*(a; m^*)}{\partial m^*} < 0 \); when \( t = \tilde{t} \), \( \frac{\partial \Psi}{\partial \bar{y}} = 0 \) and \( \frac{\partial t^*(a; m^*)}{\partial m^*} = 0 \); when \( t > \tilde{t} \), \( \frac{\partial \Psi}{\partial \bar{y}} < 0 \) and \( \frac{\partial t^*(a; m^*)}{\partial m^*} > 0 \).

Second, \( f(t; m_1^*) < f(t; m_2^*) \) when \( t < \tilde{t} \) and \( f(t; m_1^*) > f(t; m_2^*) \) when \( t > \tilde{t} \). This is straightforward as

\[
f(t; m^*) = (q_H + q_L) f_H(t) + q_L (1 - m^*) f_L(t)
\]

is increasing in \( m^\star \) when \( f^H(t) - f^L(t) > 0 \), i.e. \( t > \tilde{t} \) and decreasing in \( m^\star \) when \( f^H(t) - f^L(t) < 0 \), i.e. \( t < \tilde{t} \).

We divide the discussion into cases, depending on the magnitude of \( b \).

Case 1: when \( b \leq \Psi(\int_{f_H(t^*)}^{f_L(t^*)} dx; m^*) \), then \( G^C(x; m^*) = 0 \) and thus \( A^C(m^*) = \int_0^b G^C(x; m^*) = 0 \) which is independent of \( m^\star \).

Case 2: when \( \Psi(\int_{f_H(t^*)}^{f_L(t^*)} dx; m^*) < b \leq \Psi(\int_{f_H(t^*)}^{f_L(t^*)} dx; m^*) \), then

\[
A^C(m^*) = \int_{\Psi(\int_{f_H(t^*)}^{f_L(t^*)} dx; m^*)}^{\int_{f_H(t^*)}^{f_L(t^*)} dx} G^C(x; m^*) dx
\]

(20)

When \( \Psi(\int_{f_H(t^*)}^{f_L(t^*)} dx; m^*) < b \leq q_H \), take derivative of equation (20) with respect to \( m^* \) results
\[
\frac{\partial A^C(m^* \ | \ m^*)}{\partial m^*} = \frac{\partial}{\partial m^*} \int_0^b G^C(x; m^*) \, dx \\
= -G^C\left(\Psi\left(\frac{f_H(t_L)}{f_L(t_L)}, m^* \ | \ m^*\right), m^* \right) \cdot \frac{\partial \Psi\left(\frac{f_H(t_L)}{f_L(t_L)}, m^* \mid m^*\right)}{\partial m^*} \\
+ \int_0^b \Psi\left(\frac{f_H(t_L)}{f_L(t_L)}, m^* \mid m^*\right) \frac{\partial G^C(x; m^*)}{\partial m^*} \, dx \\
= \int_0^b \Psi\left(\frac{f_H(t_L)}{f_L(t_L)}, m^* \mid m^*\right) \frac{\partial G^C(x; m^*)}{\partial m^*} \, dx \\
= \int_0^b \Psi\left(\frac{f_H(t_L)}{f_L(t_L)}, m^* \mid m^*\right) \frac{\partial \Psi\left(\frac{f_H(t_L)}{f_L(t_L)}, m^* \mid m^*\right)}{\partial m^*} \, dx \\
= \int_0^b \Psi\left(\frac{f_H(t_L)}{f_L(t_L)}, m^* \mid m^*\right) \frac{\partial}{\partial m^*} \int_{t_L}^{t^*} f(t; m^*) \, dt \, dx \\
= \int_0^b \Psi\left(\frac{f_H(t_L)}{f_L(t_L)}, m^* \mid m^*\right) \int_{t_L}^{t^*} f(t; m^*) \, dt \, dx \\
+ \int_0^b \Psi\left(\frac{f_H(t_L)}{f_L(t_L)}, m^* \mid m^*\right) \int_{t_L}^{t^*} f(t; m^*) \, dt < 0
\] (21)

When \( b \leq q_H = \Psi\left(\frac{f_H(t_L)}{f_L(t_L)}, m^*\right) < 0 \) when \( x < b \) and \( t < t^*(x; m^*) \). Therefore both terms in equation (21) is negative and \( \frac{\partial}{\partial m^*} \int_0^b G^C(x; m^*) \, dx < 0 \).

In addition,

\[
\frac{\partial^2}{\partial b \partial m^*} A^C(m^*) \\
= \frac{\partial^2}{\partial b \partial m^*} \int_0^b G^C(x; m^*) \, dx \\
= \frac{\partial}{\partial m^*} G^C(b; m^*) \\
= \frac{\partial}{\partial m^*} \int_{t_L}^{t^*} f(t; m^*) \, dt < 0
\]

as higher \( m^* \) shifts the distribution of \( f(t; m^*) \) to the right in the sense of first-order stochastic dominance. Therefore \( \frac{\partial A(m^*)}{\partial m^*} \) is decreasing in \( b \) and thus \( \frac{\partial A(m^*)}{\partial m^*} < 0 \) when \( \Psi\left(\frac{f_H(t_L)}{f_L(t_L)}, m^*\right) < b \leq \Psi\left(\frac{f_H(t_H)}{f_L(t_H)}, m^*\right) \).
When $b > \Psi\left(\frac{f^H(T)}{f^L(T)}\right)$, $m^*$, we have

$$A^C(m^*) = \int_{\Psi\left(\frac{f^H(T)}{f^L(T)}\right)}^{b} G^C(x; m^*)dx + \int_{\Psi\left(\frac{f^H(T)}{f^L(T)}\right)}^{b} G^C(x; m^*)dx$$

$$= \Psi\left(\frac{f^H(T)}{f^L(T)}\right) G^C(x; m^*) - \int_{\Psi\left(\frac{f^H(T)}{f^L(T)}\right)}^{b} xG^C(x; m^*) + b - \Psi\left(\frac{f^H(T)}{f^L(T)}\right) m^*$$

$$= \Psi\left(\frac{f^H(T)}{f^L(T)}\right) m^* - E[\gamma(s_C, m^*)] + b - \Psi\left(\frac{f^H(T)}{f^L(T)}\right) m^*$$

where we used integration parts in arriving at the second equality and law of iterated expectations (i.e. the first informational property) in arriving at the final inequality. Clearly $A^C(m^*)$ is independent of $m^*$.

We thus have proved that $A^C(m^*) = \int_{0}^{b} G^C(x; m^*)dx$ is decreasing with $m^*$ and the decreasing is strict on a set of non-zero measure. Therefore $\gamma(s_C, m^*)$ SOSD $\gamma(s_C, m_2^*)$ whenever $m_1^* > m_2^*$.

We now prove that $\gamma(s_D, m^*)$ SOSD $\gamma(s_D, m_5^*)$ whenever $m_1^* > m_2^*$. Again we first need to explicitly express the distribution of $\gamma$, which is binary in this case. Note that for general $m^*$, $\gamma(s_D, m^*)$ is binary with

$$\gamma(l, m^*) = \frac{q_H f^H(T)}{(q_H + q_L m^*) f^H(T) + q_L (1 - m^*) f^L(T)}$$

and

$$\gamma(h, m^*) = \frac{q_H (1 - F^H(T))}{(q_H + q_L m^*) (1 - F^H(T)) + q_L (1 - m^*) (1 - F^L(T))}$$

Note that $\gamma(l, m^*) < \gamma(h, m^*)$ as $\Psi(t, m^*)$ is increasing in $t$ and $\frac{f^H(T)}{f^L(T)} < \frac{1 - F^H(T)}{1 - F^L(T)}$ as $F^H(T) < F^L(T)$ due to first-order stochastic dominance.
This implies that the cumulative distribution function of $\tilde{\gamma}(s_D, m^*)$ is

$$G^D(a) = \begin{cases} 
0 & \text{if } a < \Psi\left(\frac{F^H(T)}{F^L(T)}, m^*\right) \\
F(T; m^*) & \text{if } \Psi\left(\frac{F^H(T)}{F^L(T)}, m^*\right) \leq a < \Psi\left(\frac{1-F^H(T)}{1-F^L(T)}, m^*\right) \\
1 & \text{if } a \geq \Psi\left(\frac{1-F^H(T)}{1-F^L(T)}, m^*\right)
\end{cases}$$

where $F(T; m^*) = (q_H + q_L m^*) F^H(T) + q_L (1-m^*) F^L(T)$. 

Note that since $\frac{F^H(T)}{F^L(T)} < 1$ and $\frac{1-F^H(T)}{1-F^L(T)} > 1$, $\Psi\left(\frac{F^H(T)}{F^L(T)}, m^*\right)$ is increasing in $m^*$ and $\Psi\left(\frac{1-F^H(T)}{1-F^L(T)}, m^*\right)$ is decreasing in $m^*$. In addition, $F(T; m^*)$ is decreasing in $m^*$ as $F^H(T) < F^L(T)$. We again prove that $\tilde{\gamma}(s_D, m^*)$ SOSD $\tilde{\gamma}(s_C, m^*)$ by showing that $A^D(m^*) = \int_0^b G^D(x, m^*)dx$ is decreasing with $m^*$ and strictly decreasing over a set of non-zero measure. Again we divide the discussion into cases:

Case 1: when $b < \Psi\left(\frac{F^H(T)}{F^L(T)}, m^*\right)$, then $A^D(m^*) = 0$ and thus constant of $m^*$.

Case 2: when $\Psi\left(\frac{F^H(T)}{F^L(T)}, m^*\right) \leq b < \Psi\left(\frac{1-F^H(T)}{1-F^L(T)}, m^*\right)$, then

$$A^D(m^*) = \int_0^b G^D(x, m^*)dx$$

which is decreasing in $m^*$ as $F(T; m^*)$ is decreasing in $m^*$.

Case 3: when $b \geq \Psi\left(\frac{1-F^H(T)}{1-F^L(T)}, m^*\right)$, then

$$A^D(m^*) = \int_{\Psi\left(\frac{1-F^H(T)}{1-F^L(T)}, m^*\right)}^{\Psi\left(\frac{F^H(T)}{F^L(T)}, m^*\right)} F(T; m^*)dx + \int_b^{\Psi\left(\frac{1-F^H(T)}{1-F^L(T)}, m^*\right)} dx$$

which is independent of $m^*$.

Thus $A^D(m^*) = \int_0^b G^D(x, m^*)dx$ is weakly decreasing in $m^*$ and strictly decreasing over
a set of non-zero measure. This implies that \( \Pr(\omega = H|s_D(m_1^*)) \) second-order stochastically dominates \( \Pr(\omega = H|s_D(m_2^*)) \) when \( m_1^* > m_2^* \). 

Proof of Proposition 2:

**Proof.** Recall that for both regimes,

\[
I^*(s) = E[\omega|s; m^*] = \gamma(s; m^*)
\]

and

\[
V \equiv E[v(\omega, I^*)] = \frac{1}{2\lambda} E[\gamma^2(s; m^*)]
\]

This implies that

\[
W = \frac{\alpha}{2\lambda} E[\gamma^2(s; m^*)] + (1 - \alpha)\frac{\beta}{\lambda} E[\gamma(s; m^*)] - (1 - \alpha)\kappa K(m^*)
\]

From Lemma 1, \( E[\gamma(s; m^*)] = q_H \) under both regimes. Thus, when \( m^* \) is the same, the ranking of \( W \) is the same as the ranking of \( E[\gamma^2(s; m^*)] \). Still, from Lemma 1, \( s_C \) Blackwell dominates \( s_D \), which implies that \( s_C \) is more integral precise than \( s_D \). Thus \( \gamma(s,D,m^*) \) second-order stochastically dominates \( \gamma(s,C,m^*) \). Since \( \gamma^2(s;m^*) \) is a convex function of \( \gamma \), \( E[\gamma^2(s,D,m^*)] < E[\gamma^2(s,C,m^*)] \), implying that \( W_D < W_S \) when \( m^* \) is the same and less than 1. When \( m^* = 1 \), \( E[\gamma^2(s,D,m^*)] = E[\gamma^2(s,C,m^*)] \) and thus \( W_D = W_S \). The proof is now complete. 

Proof of Proposition 3:

**Proof.** Recall that from the proof of Proposition 2:

\[
W(m^*) = \frac{\alpha}{2\lambda} E[\gamma^2(s; m^*)] + (1 - \alpha)\frac{\beta}{\lambda} q_H - (1 - \alpha)\kappa K(m^*)
\]

where we explicitly write out the dependence on \( m^* \). From Lemma 1, \( \gamma(s; m_1^*) \) second-order stochastically dominates \( \gamma(s; m_2^*) \) whenever \( m_1^* > m_2^* \). Since \( \gamma^2(s;m^*) \) is a convex function of \( \gamma \), \( E[\gamma^2(s;m^*)] \) decreases with \( m^* \). Since \( K(m^*) \) increase with \( m^* \), \( W(m^*) \) decrease with \( m^* \) when \( m^* < 1 \).

Proof of Proposition 4:

**Proof.** Recall that \( m_1^* \) and \( m_2^* \) are characterized by first-order conditions, i.e. equation (14), which we rewrite here after rearranging terms.

\[
q_L \int \gamma(s_C,m_1^*)[f^H(s_C) - f^L(s_C)]ds_C - \frac{\kappa \lambda}{\beta} K'(m_1^*) = 0 \tag{22}
\]

and

\[
q_L \int \gamma(s_D,m_2^*)[f^H(s_D) - f^L(s_D)]ds - \frac{\kappa \lambda}{\beta} K'(m_2^*) = 0 \tag{23}
\]
Now we prove \( m^*_C > m^*_D \) by contradiction. Suppose \( m^*_C \leq m^*_D < 1 \). We thus have
\[
m^*_C - m^*_D \propto \frac{\kappa \lambda}{\beta q_L} (K'(m^*_C) - K'(m^*_D))
\]
\[
= \int \gamma(s_C, m^*_C) [f^H(s_C) - f^L(s_C)] \, ds_C - \int \gamma(s_D, m^*_D) [f^H(s_D) - f^L(s_D)] \, ds_D
\]
\[
\geq \int \gamma(s_C, m^*_D) [f^H(s_C) - f^L(s_C)] \, ds_C - \int \gamma(s_D, m^*_D) [f^H(s_D) - f^L(s_D)] \, ds_D
\]
\[
= \frac{\lambda}{\beta} \left\{ [\Pi^H_C(m^*_D) - \Pi^L_C(m^*_D)] - [\Pi^H_D(m^*_D) - \Pi^L_D(m^*_D)] \right\}
\]
\[
= \frac{\lambda}{\beta} \left\{ [\Pi^H_C(m^*_D) - \Pi^H_D(m^*_D)] - [\Pi^L_C(m^*_D) - \Pi^L_D(m^*_D)] \right\} > 0
\] (24)
resulting in contradiction.

The first inequality is because \( \int \gamma(s_C, m^*) [f^H(s_C) - f^L(s_C)] \, ds_C = \frac{\lambda}{\beta} [\Pi^H_C(m^*) - \Pi^L_C(m^*)] \) is decreasing in \( m^* \), as shown in the proof of Proposition 1.

The last inequality is proved as follows. Note that we can rewrite \( V(m) \) as
\[
V(m) = \frac{1}{2\lambda} E[\gamma^2(s; m)]
\]
\[
= \frac{1}{2\lambda} \int \gamma^2(s; m) g(s; m) \, ds
\]
\[
= \frac{1}{2\lambda} \int \gamma(s; m) \frac{q_H g^H(s)}{g(s; m)} g(s; m) \, ds
\]
\[
= \frac{q_H}{2\lambda} \int \gamma(s; m) g^H(s) \, ds
\]
\[
= \frac{q_H}{2\beta} \Pi^H(m)
\]
Therefore
\[
\Pi^H_C(m^*_D) - \Pi^H_D(m^*_D) = \frac{2\beta}{q_H} [V_C(m^*_D) - V_D(m^*_D)] > 0
\]
as \( V_C(m^*_D) > V_D(m^*_D) \) from Proposition 2. In addition, law of iterated expectations imply that for \( i = C, D \):
\[
E[I^*_i] = \frac{1}{\lambda} E[\gamma(s_i; m^*_i)] = \frac{q_H}{\lambda}
\]
This also implies that
\[
U(m^*_i; m^*_i) + \kappa K(m^*_i) = \beta \frac{q_H}{\lambda}
\] (25)
where \( U \) is defined as in equation (16). Following equation (11), we can express equation (25) as
\[
q_H \Pi^H_i(m^*_C(\kappa^*)) + q_L \Pi^L_i(m^*_C(\kappa^*)) + q_L m^*_i(\kappa^*) (\Pi^H_i(m^*_C(\kappa^*)) - \Pi^L_i(m^*_C(\kappa^*)) = \beta \frac{q_H}{\lambda}
\]
This implies that
\[
q_H \Pi^H_C(m^*_C(\kappa^*)) + q_L \Pi^L_C(m^*_C(\kappa^*)) + q_L m^*_C(\kappa^*) (\Pi^H_C(m^*_C(\kappa^*)) - \Pi^L_C(m^*_C(\kappa^*)) = q_H \Pi^H_D(m^*_D(\kappa^*)) + q_L \Pi^L_D(m^*_D(\kappa^*)) + q_L m^*_D(\kappa^*) (\Pi^H_D(m^*_D(\kappa^*)) - \Pi^L_D(m^*_D(\kappa^*))
\]
Thus, after rearranging terms, results in
\[(q_H + q_Lm_D^*)[\Pi_C^H(m_D^*) - \Pi_D^H(m_D^*)] + q_L(1 - m_D^*)[\Pi_C^L(m_D^*) - \Pi_D^L(m_D^*)] = 0\]
Thus \(\Pi_C^H(m_D^*) - \Pi_D^H(m_D^*) > 0\) implies that \(\Pi_C^L(m_D^*) - \Pi_D^L(m_D^*) < 0\).

Proof of Theorem 1:

Proof. Note that for \(i = C, D\),

\[
W_i(\kappa) = \alpha V_i(\kappa) + (1 - \alpha)U_i(\kappa)
\]
\[
= \alpha V_i(\kappa) + (1 - \alpha)\beta(q_H - (1 - \alpha)\kappa K(m_i^*(\kappa)))
\]
\[
= (1 - \alpha)\frac{\beta}{\lambda}q_H + \alpha[V_i(\kappa) - \frac{1 - \alpha}{\alpha}\kappa K(m_i^*(\kappa))]
\]
\[
= (1 - \alpha)\frac{\beta}{\lambda}q_H + \alpha[q_H^H(m_i^*(\kappa)) - \frac{1 - \alpha}{\alpha}\kappa K(m_i^*(\kappa))]
\]

where we explicitly write the dependence of \(m^*, V, U\) and \(W\) on \(\kappa\). The last inequality is derived in the proof of Proposition 4.

Since \((1 - \alpha)\frac{\beta}{\lambda}q_H\) is a constant of \(\kappa\), comparing \(W(\kappa)\) across regimes is equivalent to comparing \(\frac{q_H}{2\beta}\Pi_i^H(m_i^*(\kappa)) - \frac{1 - \alpha}{\alpha}\kappa K(m_i^*(\kappa))\), which we denote as \(\varphi_i(\kappa)\), i.e.

\[
\varphi_i(\kappa) \equiv \frac{q_H}{2\beta}\Pi_i^H(m_i^*(\kappa)) - \frac{1 - \alpha}{\alpha}\kappa K(m_i^*(\kappa))
\]

When \(\kappa\) is sufficiently small, \(m_C^* = 1 > m_D^*\), resulting in \(\varphi_C(1) - \varphi_D(m_D^*) = \frac{q_H}{2\beta}[\Pi_C^H(1) - \Pi_D^H(m_D^*)] - \frac{1 - \alpha}{\alpha}\kappa[K(1) - K(m_D^*(\kappa))] < 0\) as \(\Pi_C^H(1) < \Pi_D^H(m_D^*(\kappa))\) and \(K(1) > K(m_D^*(\kappa))\). To see why when \(\kappa\) is sufficiently small, \(m_C^* = 1\), note that when \(m^* = 1\), in the continuous regime, \(\gamma(s_C, 1) = q_H\) when \(s_C = t \in [\bar{t}_H, \bar{t}_H]\) and the off-equilibrium belief is that \(\gamma(s_C, 1) = 0\) when \(s_C = t \in [\underline{t}_H, \bar{t}_H]\). Thus,

\[
\Pi_C^H(1) - \Pi_C^L(1)
\]
\[
= \frac{\beta}{\lambda} \int_{\underline{t}_H}^{\bar{t}_H} \gamma(s_C, 1)[g_H(s_C) - g_L(s_C)]ds_C
\]
\[
= q_H \frac{\beta}{\lambda} G_L(\bar{t}_H)
\]

Recall from the proof of Proposition 1 that

\[
\Omega(1) = q_L(\Pi_C^H(1) - \Pi_C^L(1)) - \kappa K'(1)
\]

Thus, in the continuous regime, if \(\kappa\) is sufficiently small so that \(\kappa K'(1) \leq q_L q_H \frac{\beta}{\lambda}(\bar{t}_H - \underline{t}_L)\), then \(\Omega_C(1) \geq 0\) and \(m_C^* = 1\).

To see why \(m_D^* < 1\), note that in the recognition regime, since \(T \in [\underline{t}_H, \bar{t}_L]\), \(\gamma(s_D, 1) = q_H\)
when $s_D = h, l$. Therefore
\[ \Pi_D^H(1) = \sum_{s_D} \gamma(s_D, 1)[g^H(s_D) - g^L(s_D)]ds_D \]
\[ = 0, \]
resulting in $\Omega_D(1) < 0$ and thus $m_D^* < 1$.

When $\kappa \rightarrow +\infty$, both $m_C^*$ and $m_D^* \rightarrow 0$. Thus
\[ \varphi_C(0) - \varphi_D(0) \]
\[ \rightarrow \frac{q_H}{2\beta}[\Pi_C^H(0) - \Pi_D^H(0)] - \frac{1 - \alpha}{\alpha} \lim_{\kappa \rightarrow +\infty} \kappa[K(m_C^*(\kappa)) - K(m_D^*(\kappa))] \]
\[ = \frac{q_H}{2\beta}[\Pi_C^H(0) - \Pi_D^H(0)] \]
\[ > 0 \]
as $\Pi_C^H(0) - \Pi_D^H(0) > 0$ from Proposition 2.

To see why the last equality holds, note that when $\kappa \rightarrow +\infty$, $m_C^*$ and $m_D^* \rightarrow 0$. The first-order conditions result in $\frac{\kappa}{qL}K'(m_C^*(\kappa)) \rightarrow \Pi_C^H(0) - \Pi_C^L(0)$ and $\frac{\kappa}{qL}K'(m_D^*(\kappa)) \rightarrow \Pi_D^H(0) - \Pi_D^L(0)$. Thus,
\[ \lim_{\kappa \rightarrow +\infty} \kappa K(m_C^*(\kappa)) \]
\[ = \lim_{\kappa \rightarrow +\infty} \frac{\kappa K'(m_C^*(\kappa))}{K'(m_C^*(\kappa))} \]
\[ = qL[\Pi_C^H(0) - \Pi_C^L(0)] \lim_{\kappa \rightarrow +\infty} \frac{K(m)}{K'(m)} \]
\[ = qL[\Pi_C^H(0) - \Pi_C^L(0)] \frac{1}{0} \]
\[ = 0 \]
as the last equality follows from L’Hospital’s rule, i.e. $\lim_{m \rightarrow 0} \frac{K(m)}{K'(m)} = \lim_{m \rightarrow 0} \frac{K'(m)}{K''(m)} = 0$.

We can similarly prove that $\lim_{\kappa \rightarrow +\infty} \kappa K(m_D^*(\kappa)) = 0$, resulting in $\lim_{\kappa \rightarrow +\infty} \kappa[K(m_C^*(\kappa)) - K(m_D^*(\kappa))] = 0$.

Thus by intermediate value theorem there exists at least one value of $\kappa$, denoted as $\tilde{\kappa}$ s.t. $\varphi_C(\tilde{\kappa}) - \varphi_D(\tilde{\kappa}) = 0$ and $\varphi_C(\kappa) > \varphi_D(\kappa)$ if $\kappa$ is larger and around $\tilde{\kappa}$ and $\varphi_C(\kappa) < \varphi_D(\kappa)$ if $\kappa$ is smaller and around $\tilde{\kappa}$. We will delegate the proof that $\tilde{\kappa}$ is unique towards the end as it is quite technical. We first derive the comparative statics of $\tilde{\kappa}$ by assuming that it is unique.

Recall that $\tilde{\kappa}$ is defined by
\[ \phi(\tilde{\kappa}) \equiv \Lambda(m_D^*(\tilde{\kappa})) - \Lambda(m_C^*(\tilde{\kappa})) = 0 \quad (26) \]
where $\Lambda(m) \equiv (1 - m)K'(m) - \frac{(1 - \alpha)}{\alpha}K(m)$.

Take the derivative of both sides of equation (26) with respect to $\lambda$ results in, as is the implication from the implicit function theorem,
\[ \frac{\partial \tilde{\kappa}}{\partial \lambda} = -\frac{\partial \phi(\tilde{\kappa})}{\partial \lambda} \left( \frac{\partial \phi(\tilde{\kappa})}{\partial \tilde{\kappa}} \right) \]
Let’s now focus on the signs of the two terms, $\frac{\partial \phi(\pi)}{\partial \kappa}$ and $\frac{\partial \phi(\pi)}{\partial \beta}$, respectively. Note that

$$\frac{\partial \phi(\pi)}{\partial \kappa} = \Lambda’((m^*_D(\pi))) \frac{\partial m^*_D(\pi)}{\partial \kappa} |_{\kappa = \pi} - \Lambda’((m^*_C(\pi))) \frac{\partial m^*_C(\pi)}{\partial \kappa} |_{\kappa = \pi}$$

As will be discussed below in the proof of the uniqueness of $\pi$, $\Lambda’((m^*_D(\pi))) > 0$ and $\Lambda’((m^*_C(\pi))) < 0$. In addition, from Proposition 1, $\frac{\partial m^*_D(\pi)}{\partial \kappa} |_{\kappa = \pi} < 0$ and $\frac{\partial m^*_C(\pi)}{\partial \kappa} |_{\kappa = \pi} < 0$. This results in $\frac{\partial \phi(\pi)}{\partial \kappa} < 0$.

In addition,

$$\frac{\partial \phi(\pi)}{\partial \beta} = \Lambda’((m^*_D(\pi))) \frac{\partial m^*_D(\pi)}{\partial \beta} - \Lambda’((m^*_C(\pi))) \frac{\partial m^*_C(\pi)}{\partial \beta}$$

As will be discussed below in the proof of the uniqueness of $\pi$, $\Lambda’((m^*_D(\pi))) > 0$ and $\Lambda’((m^*_C(\pi))) < 0$. In addition, from Proposition 1, $\frac{\partial m^*_D(\pi)}{\partial \beta} > 0$ and $\frac{\partial m^*_C(\pi)}{\partial \beta} > 0$. This results in $\frac{\partial \phi(\pi)}{\partial \beta} > 0$.

Similarly, using the implicit function theorem and take the derivative of both sides of equation (26) with respect to $\beta$ results in

$$\frac{\partial \pi}{\partial \beta} = -\frac{\frac{\partial \phi(\pi)}{\partial \beta}}{\frac{\partial \phi(\pi)}{\partial \kappa}}$$

Note that $\frac{\partial \phi(\pi)}{\partial \kappa} < 0$ as shown above. In addition,

$$\frac{\partial \phi(\pi)}{\partial \beta} = \Lambda’((m^*_D(\pi))) \frac{\partial m^*_D(\pi)}{\partial \beta} - \Lambda’((m^*_C(\pi))) \frac{\partial m^*_C(\pi)}{\partial \beta}$$

Again, it will be shown that $\Lambda’((m^*_D(\pi))) > 0$ and $\Lambda’((m^*_C(\pi))) < 0$. In addition, from Proposition 1, $\frac{\partial m^*_D(\pi)}{\partial \beta} > 0$ and $\frac{\partial m^*_C(\pi)}{\partial \beta} > 0$. This results in $\frac{\partial \phi(\pi)}{\partial \beta} > 0$. Therefore $\frac{\partial \pi}{\partial \beta} > 0$.

To complete the proof we now show that such $\pi$ is unique. To see this, note that law of iterated expectations implies that for $i = C, D$

$$q_H = E[\gamma(s_i; m^*_i)]$$

which in turn implies that

$$U(m^*_i; m^*_i) + \kappa K(m^*_i) = \frac{\beta}{\lambda} q_H$$

Using the definition of $U$ from equation (11), we can write out equation (27) as

$$q_H \Pi^H_i(m^*_i) + q_L \Pi^L_i(m^*_i) + q_L m^*_i (\Pi^H_i(m^*_i) - \Pi^L_i(m^*_i)) = \frac{\beta}{\lambda} q_H$$

(28)

In addition, the first-order condition, i.e. equation (12) implies that

$$q_L (\Pi^H_i(m^*_i) - \Pi^L_i(m^*_i)) = \kappa K'(m^*_i)$$

and that

$$\Pi^L_i(m^*_i) = \Pi^H_i(m^*_i) - \frac{\kappa K'(m^*_i)}{q_L}$$
Insert into equation (28) results in

\[
q_H \Pi_i^H(m_i^*) + q_L(\Pi_i^H(m_i^*) - \frac{\kappa K'(m_i^*)}{q_L}) + m_i^* \kappa K'(m_i^*) = \frac{\beta}{\lambda}q_H
\]

\[
\Leftrightarrow \Pi_i^H(m_i^*) = \frac{\beta}{\lambda}q_H + (1 - m_i^*) \kappa K'(m_i^*)
\]

This results in

\[
\varphi_i(\kappa) = \frac{q_H}{2\beta} \Pi_i^H(m_i^*(\kappa)) - \frac{1 - \alpha}{\alpha} \kappa K(m_i^*(\kappa))
\]

\[
= \frac{(q_H)^2}{2\lambda} + \frac{q_H}{2\beta} (1 - m_i^*) \kappa K'(m_i^*(\kappa)) - \frac{1 - \alpha}{\alpha} \kappa K(m_i^*(\kappa))
\]

Thus, \( \varphi_C(\kappa^*) = \varphi_D(\kappa^*) \) is equivalent to

\[
\frac{q_H}{2\beta} (1 - m_C^*(\kappa^*)) K'(m_C^*(\kappa^*)) - \frac{1 - \alpha}{\alpha} K(m_C^*(\kappa^*))
\]

\[
= \frac{q_H}{2\beta} (1 - m_D^*(\kappa^*)) K'(m_D^*(\kappa^*)) - \frac{1 - \alpha}{\alpha} K(m_D^*(\kappa^*))
\]

\[
\Leftrightarrow \Lambda(m_C^*(\kappa^*)) = \Lambda(m_D^*(\kappa^*))
\]

where \( \Lambda(m) \equiv \frac{q_H}{2\beta} (1 - m) K'(m) - \frac{1 - \alpha}{\alpha} K(m) \). We want to show that there is a unique \( \bar{\kappa} \) that satisfies equation (29). Figure ?? graphically shows how we prove the uniqueness of \( \bar{\kappa} \). Note that the purple unbroken line in the figure is an example of the graph of \( \Lambda(m) \) when \( \alpha = 1 \). The assumptions on \( K(m) \) implies that \( \Lambda(m) \) first increases with \( m \), then reaches a maximum and then decreases with \( m \). Equation (29) implies there is a \( \bar{\kappa} \) such that \( \Lambda(m_C^*(\bar{\kappa})) = \Lambda(m_D^*(\bar{\kappa})) \). Suppose that the solution of equation (29) is not unique, then there are at least two distinct solutions, \( \bar{\kappa}_1 \) and \( \bar{\kappa}_2 \), corresponding to the two horizontal dashed lines in the figure. Since \( m_C^*(\bar{\kappa}_1) > m_D^*(\bar{\kappa}_1) \), the figure then implies that it must be the case that \( m_D^*(\bar{\kappa}_2) < m_D^*(\bar{\kappa}_1) \) and \( m_C^*(\bar{\kappa}_2) > m_C^*(\bar{\kappa}_1) \). However, since both \( m_C^* \) and \( m_D^* \) are decreasing in \( \kappa \), \( m_D^*(\bar{\kappa}_2) < m_D^*(\bar{\kappa}_1) \) implies \( \bar{\kappa}_2 > \bar{\kappa}_1 \), which in turn implies that \( m_C^*(\bar{\kappa}_2) > m_C^*(\bar{\kappa}_1) \), resulting in a contradiction. The rest of the proof is merely showing the illustrated intuition in rigorous algebra.

First note that \( \Lambda(0) = 0 \) and \( \Lambda(1) \leq 0 \). In addition, \( \Lambda'(m) = K''(m) - \frac{1}{\alpha} K'(m) > 0 \) when \( m \) is sufficiently close to 0 as \( \lim_{m \to 0} \frac{K'(m)}{K''(m)} = 0 \). To see this, note that \( \lim_{m \to 0} \frac{K'(m)}{K''(m)} = 0 \) implies \( \forall \varepsilon > 0, \exists m(\varepsilon) \) s.t. \( \frac{K'(m)}{K''(m)} < \varepsilon \), or, equivalently, \( K''(m) > \frac{K'(m)}{\varepsilon} \). Pick \( 0 < \varepsilon < \frac{1}{1 + \frac{1}{\alpha}} \). Then when \( 0 < m < m(\varepsilon) \), \( K''(m) > \frac{K'(m)}{\varepsilon} > [1 + \frac{1}{\alpha}] K'(m) \), resulting in \( \Lambda'(m) > 0 \) when \( 0 < m < m(\varepsilon) \). Thus, \( \Lambda(m) \) will reach (possibly more than one) maximum when \( m \in (0,1) \). A necessary condition for an interior maximum is the satisfaction of the first-order condition, i.e. \( \Lambda'(m) = \frac{\beta}{\lambda}q_H + (1 - m^*) \kappa K'(m^*) \). Note that this does not mean that the second order derivative of \( \Lambda(m) \) is always negative. It implies a weaker condition that the second order derivative of \( \Lambda(m) \) crosses zero only once.
0, or, equivalently,

\[ 1 - m = [1 + \frac{(1 - \alpha)}{\alpha} \frac{K'(m)}{K''(m)}] \]  

(30)

will have at least one solution on \((0, 1)\). Note that

\[
\frac{\partial}{\partial m} \left( \frac{K'(m)}{K''(m)} \right) = \frac{(K''(m))^2 - K'(m)K'''(m)}{(K''(m))^2},
\]

thus \(\frac{K'(m)}{K''(m)}\) is weakly increasing in \(m\) if and only if \((K''(m))^2 - K'(m)K'''(m) \geq 0\). Since

\(1 - m\) is strictly decreasing in \(m\) whereas \(\frac{K'(m)}{K''(m)}\) is weakly increasing in \(m\), there is a unique \(\overline{m} \in (0, 1)\) that satisfies equation (30), i.e. \(\Lambda'(\overline{m}) = 0\). Since \(\overline{m}\) is the only solution of the first-order condition, \(\Lambda(m)\) reaches its maximum at \(\overline{m}\). This also implies that \(\Lambda'(m) > 0\) when \(m \in (0, \overline{m})\) and that \(\Lambda'(m) < 0\) when \(m \in (\overline{m}, 1)\). To see why this is true, suppose that \(\Lambda'(m) \leq 0\) for some \(\overline{m} \in (0, \overline{m})\). Clearly \(\Lambda'(\overline{m}) < 0\) as \(\Lambda'(\overline{m}) = 0\) would violate the condition that \(\overline{m}\) is the unique solution that satisfies equation (30). When \(\Lambda'(\overline{m}) < 0\), there must exist at least one \(m \in (\overline{m}, \overline{m})\) such that \(\Lambda(m) > 0\) as if otherwise \(\Lambda'(m) < 0\) for all \(m \in (\overline{m}, \overline{m})\), then \(\Lambda(m) > \Lambda(\overline{m})\), violating that \(\Lambda(m)\) reaches its maximum at \(\overline{m}\). Now suppose that \(\Lambda(m) > 0\) and \(\Lambda'(\overline{m}) < 0\), then by intermediate value theorem there must exist a \(\overline{m} \in (\overline{m}, \overline{m})\) such that \(\Lambda'(\overline{m}) = 0\). Since \(\overline{m} \leq \overline{m}\), it is in contradiction with \(\overline{m}\) being the unique solution that satisfies equation (30). We thus have proved that \(\Lambda'(m) > 0\) when \(m \in (0, \overline{m})\). The proof that \(\Lambda'(m) < 0\) when \(m \in (\overline{m}, 1)\) is essentially the same and thus omitted.

Note that since \(\Lambda(m_C^*(\kappa^*)) = \Lambda(m_D^*(\kappa^*))\), \(\Lambda'(m) > 0\) when \(m \in (0, \overline{m})\) and that \(\Lambda'(m) < 0\) when \(m \in (\overline{m}, 1)\) and that \(m_C^*(\kappa^*) > m_D^*(\kappa^*)\), it must be the case that \(m_C^*(\kappa^*) > \overline{m} > m_D^*(\kappa^*)\). Suppose the solution to equation (30) is not unique, then there exists \(\overline{\kappa}_1 \neq \overline{\kappa}_2\) s.t.

\[ \Lambda(m_C^*(\overline{\kappa}_1)) = \Lambda(m_D^*(\overline{\kappa}_1)) \]

and

\[ \Lambda(m_C^*(\overline{\kappa}_2)) = \Lambda(m_D^*(\overline{\kappa}_2)) \]

Without loss of generality assume that \(\overline{\kappa}_1 < \overline{\kappa}_2\). Then \(m_C^*(\overline{\kappa}_1) > m_C^*(\overline{\kappa}_2) > \overline{m}\) and \(\overline{m} > m_D^*(\overline{\kappa}_1) > m_D^*(\overline{\kappa}_2)\) as we show above that \(\frac{\partial m_C^*}{\partial \kappa} < 0\) and \(\frac{\partial m_D^*}{\partial \kappa} < 0\). Since \(\Lambda'(m) > 0\) when \(m \in (0, \overline{m})\) and that \(\Lambda'(m) < 0\) when \(m \in (\overline{m}, 1)\), we have

\[ \Lambda(m_C^*(\overline{\kappa}_1)) = \Lambda(m_D^*(\overline{\kappa}_1)) > \Lambda(m_D^*(\overline{\kappa}_2)) = \Lambda(m_C^*(\overline{\kappa}_2)) \]

resulting in a contradiction. Therefore there is a unique \(\overline{\kappa}\) that satisfies equation (29) and \(W_C > W_D\) if and only if \(\kappa < \overline{\kappa}\). 

\[ \blacksquare \]

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