Information Production and Market Feedback

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Abstract

We analyze the joint information acquisition problem of a firm manager and traders in financial markets, when the firm manager conditions her investment decisions on information revealed through stock prices and the firm is exposed to multiple sources of uncertainty. We highlight a fundamental mismatch: while traders want to collect the same information as the manager to maximize trading profits, the manager optimally diversifies her information sources and tries to acquire orthogonal information. Due to this coordination problem, the manager relies too much on market information in equilibrium and the firm is better off hiring a dismissive manager who underestimates the informational content of prices. We also emphasize a discrepancy between market efficiency and real efficiency and show that, for instance, the former can be maximized when the latter is minimized.

Keywords: feedback effect; information acquisition; information diversity; real efficiency; price informativeness.

JEL Classification: D83, G14, G31.

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1 Introduction

It has long been recognized that the informational content of stock prices ("market efficiency") relies crucially on the willingness of traders to acquire costly information [see e.g., Grossman and Stiglitz, 1980; Verrecchia, 1982]. Similarly, it has been recognized that the profitability of firm managers' capital investment decisions ("real efficiency") depends on the quality of their private information and their ability to select optimal projects [see e.g., Lambert, 1986; Harris and Raviv, 1996]. A more recent literature argues that there might be a connection between these two efficiency concepts because some of the information in prices might be useful for managers and can help them to invest more efficiently. This "feedback effect" [Bond et al., 2012] has received significant support from the recent empirical literature [see e.g., Luo, 2005; Chen et al., 2007; Foucault and Fresard, 2012].

While existing theoretical work has focussed on the information acquisition decision of either traders or firm managers in separation, our model allows both types to acquire information simultaneously. In particular, we show that the information acquisition decision of one type generates spill-overs for the other type. On the one hand, traders’ acquisition of private information renders the equilibrium stock price more informative and allows the firm manager to extract valuable information about the investment opportunities. On the other hand, the manager’s acquisition of private information is reflected in a more efficient investment decision, which affects the firm's future value and thus the traders’ payoffs.

We study this joint information acquisition problem in a setting with multiple sources of uncertainty. More specifically, a firm manager can invest in two independent investment projects. The return on both projects is determined by different fundamentals (or "shocks") and all agents, traders and the firm manager, are ex ante uninformed about these shocks and not sure whether it is worthwhile to invest in the projects ex post. Both types can acquire private signals about these shocks but it is too costly for them to collect perfect information along both dimensions. Thus, all traders and the firm manager have to decide how to spend their limited resources most efficiently.

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1See also Edmans et al. [2012], Foucault and Fresard [2014], Edmans et al. [2017], and Dessaint et al. [2019] for empirical evidence of market feedback.
and what type of uncertainty they would like to reduce privately.

Our main insight is that this two-way interaction entails a fundamental tension of incentives. On the one hand, the firm manager can most efficiently learn from the stock price if all traders acquire private information about the project she is not acquiring private information about herself. This way the manager can rely on price information regarding the investment decision for one project and on private information for the other project. Hence, the market’s information choice is a *strategic substitute* from the manager’s perspective. On the other hand, each individual trader has an incentive to mimic the manager’s information choice and acquire information about the same shock. This way traders maximize their trading profits because they can forecast a more significant portion of the future payoff with their private signal. Hence, the manager’s information choice is a *strategic complement* for individual traders.

This fundamental mismatch in the incentives to acquire information leads to a very nuanced equilibrium information choice. More specifically, we show that the traders’ and the manager’s information choice depends on two model parameters, the traders’ information precision capacity (relative to that of the manager) and the relative payoff of the two projects which we interpret as a measure of diversity. If one of the two projects offers a particularly high payoff and the precision capacity is sufficiently small, all traders acquire information about this project. In this case, the firm manager also learns about this shock because this project offers a sufficiently high return to offset the strategic substitutability effect mentioned earlier. If, however, the traders’ information capacity is sufficiently high and the projects offer a more similar return, there is no pure-strategy equilibrium and all agents randomize between both shocks. In particular, the firm manager diversifies her information choice almost equally between both projects and acquires slightly more information about the less-profitable project. In response, traders acquire more information about the more-profitable project.

We abstract from any agency conflicts between the firm and the manager such that she invests in either project if the expected net present value is positive. In our model the manager’s expectation comprises two types of signal. First, a private signal about one of the two projects and second, a
public signal based on the firm’s stock price. In particular, we assume that the manager cannot acquire a perfect signal about both projects’ returns such that she has to rely on price information in some circumstances. As mentioned before, traders acquire more information about the more-profitable project which renders the price more informative along this dimension. Moreover, we show that a particularly high stock price represents good news about both project, while a slightly above-average price indicates that the return on the more-profitable project is high. In both cases, the manager can infer the return on at least one of the projects and invest.

More generally, the firm’s equilibrium stock price is a step function of total order flow, i.e. the sum of informed traders’ demand and that of liquidity traders. A competitive market maker observes total order flow and sets the stock price equal to the expected firm value. Importantly, the market maker can more easily interpret movements in total order flow if all traders acquire information about the same project. In this case, particularly high values reveal a "high" project fundamental and vice versa for low values. If, however, traders pursue a mixed-strategy, total order flow is affected by both fundamentals and it is not clear whether a slightly above-average order flow is the result of a "high" fundamental for project one and a "low" fundamental for project two, or vice versa. Thus, the traders’ information mix has a direct effect on the informational content of total order flow and the equilibrium stock price.

Our main focus is on the implications of information acquisition on two efficiency measures. First, we consider market efficiency which is defined as the reduction in payoff variance that can be achieved by conditioning on the stock price. This measure represents the degree to which asset prices predict future payoffs and is widely used in the empirical literature to gauge the informational content of stock prices. Second, we consider real efficiency which is defined as the firm’s expected long-run value. This measure reflects the efficiency of the firm manager’s investment decision in the two risky projects. The conventional view is that market efficiency is a good proxy for real efficiency because a higher price-payoff correlation is expected to reveal more information to real-decision makers. One of our main contributions is to show that this conventional wisdom is generally incorrect.
For example, we show that real efficiency is always maximized if diversity is high and both projects offer the same return. In this case, the degree of information overlap is minimized and the firm can learn quite efficiently from the stock price. However, we also show that this scenario can be detrimental for market efficiency. Most interestingly, market efficiency can be maximized when diversity is relatively low and real efficiency is minimized. The intuition for this finding is that the degree of overlap on the traders’ and the manager’s type of information is higher in this case which increases the price-payoff correlation but lowers investment efficiency.

The fundamental insight that the type of information in stock prices matters for market feedback is not new. Following Bond et al. [2012], who differentiate between forecasting price efficiency (FPE) and revelatory price efficiency (RPE), several empirical papers have tried to separate the two concepts [see e.g., Bai et al., 2016; Edmans et al., 2017]. We provide a formal framework that emphasizes a fundamental misalignment of incentives behind the information acquisition decisions that give rise to these efficiency measures.

In an extension of the main model, we allow the firm to hire a manager who underestimates the traders’ ability to predict firm fundamentals. We show that the firm is always better off hiring a dismissive manager because this friction mitigates the coordination problem between the manager and traders. Intuitively, a dismissive manager pays more attention to the more profitable project, which, in turn, incentives traders to acquire more information about the same project. This information acquisition equilibrium leads to more efficient capital investment decisions because it renders the price a clearer signal of the more profitable project.

The model makes three important assumptions. First, we consider a firm affected by multiple shocks that govern the return on the different investment opportunities. Potential examples of these different dimensions of uncertainty are multinational firms that are exposed to different country-level shocks or conglomerates that are exposed to different industry-level shocks. More generally, a firm might have the opportunity to launch several new products and faces uncertainty about the respective profitability. Second, we allow the firm manager and, more importantly, traders to acquire private information about the same set of fundamental shocks. Therefore,

Goldstein and Yang [2015] discuss the importance of studying different dimensions of uncertainty in a model without market feedback.
we do not preclude the trader from certain types of shocks and consider all of them learnable. Thereby we are able to ascertain the equilibrium information acquisition decision of the different types with as little restrictions as possible. Our third assumption is that neither the manager nor informed traders have sufficient resources to collect perfect information about all shocks. This assumption is important for two reasons. First, it implies that the manager can learn additional information from the stock price. Second, it also creates a trade-off for the two types and renders the information acquisition decision non-trivial. The existing literature has highlighted several frictions (like limited attention or resources) that might lead to such a constraint [see e.g., Aghion and Stein, 2008; Mondria, 2010; Kacperczyk et al., 2016, among others].

Our papers contributes to two strands of the literature. First, it is related to the literature studying the real effects of financial markets, where trading and prices affect the firms’ investment decisions, which in turn affect the firms’ cash flows. This is known as the "feedback effect" and Bond et al. [2012] provide a review of this literature. The first theoretical contributions take the agents’ information endowment as given and study market feedback with respect to a single fundamental. Building on these models, Dow et al. [2017] and Gao and Liang [2013] endogenize the traders’ signal precision in a single-shock setting, while Goldstein and Yang [2019] consider a setting with fixed information endowments but two sources of uncertainty. In contrast to these papers, we endogenize the information endowment of the manager and traders. As a result, we get several novel predictions relative to the existing work. In particular, our framework leads to a novel distinction between market efficiency and real efficiency and emphasizes the fact that it matters what kind of information financial market participants collect. In related work Benhabib et al. [2018] also study a feedback model with mutual learning by managers and traders. However, in contrast to our work, they assume that both types acquire information about different shocks. As a result, they do not focus on the strategic coordination between the manager and traders and the efficiency implications of this interaction which lies at the core of our paper.

Second, our paper is also related to the economics and finance literature studying strategic
information acquisition with multiple sources of uncertainty. Goldstein and Yang [2015] study a trading model in the spirit of Grossman and Stiglitz [1980] and show that the presence of multiple sources of uncertainty gives rise to strategic complementarities. van Nieuwerburgh and Veldkamp [2010], van Nieuwerburgh and Veldkamp [2010], and Kacperczyk et al. [2016] study trading models with multiple pieces of uncertainty and traders with limited attention capacity. Goldman [2004] and Goldman [2005] study a setting featuring a multi-division firm and endogenous information acquisition by traders. Our contribution relative to this literature is twofold. First, we allow for a real effect of the information collected by traders because of the feedback effect. Second, we also study the simultaneous information acquisition on the real side and its repercussions to the financial market. As a result, none of these papers addresses the relationship between market efficiency and real efficiency.

The remainder of the paper is organized as follows. In Section 2 we provide a description of the model. Section 3 characterizes the equilibrium outcome. Section 4 studies an extension in which the firm manager is skeptical of the traders’ signal precision. Section 5 concludes and all proofs are contained in Appendix A.

2 Model Setup

The model considers three dates, $t \in \{0, 1, 2\}$, and a single firm. The firm’s stock is traded in a secondary financial market and a manager (“she”) can increase the firm’s value through investment in two uncertain growth opportunities. The financial market is populated by informed traders, uninformed noise traders, and a competitive market maker (“he”). At $t = 0$, the firm’s manager and informed traders make an information acquisition decision and receive information about one of the two growth opportunities. At $t = 1$, trading in the financial market occurs and subsequently the firm manager decides on investment in the two growth opportunities. The manager’s decision may be influenced by the realization of the stock price which creates a feedback effect [Bond et al., 2012]. At $t = 2$, the firm’s terminal value $V$ is realized and paid out as a liquidating dividend. Figure 1 provides a timeline for the key events of the model and Appendix A.1 summarizes the
notation.

<table>
<thead>
<tr>
<th>t = 0</th>
<th>t = 1</th>
<th>t = 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>manager acquires signal $\sigma_m$</td>
<td>informed traders submit market orders $s_i$</td>
<td>firm value $V$ realized</td>
</tr>
</tbody>
</table>
| trader $i$ acquires signal $\sigma_i$ | market maker sets stock price $P$ | \[ t = x_j \]

manager chooses investments $K_j$

Figure 1: Timeline for the main model.

2.1 The Firm’s Decision

In our model, the firm is operated by a benevolent manager who maximizes the firm’s expected long-term value $V$. The firm has access to two growth opportunities, $A$ and $B$, and can choose to invest one unit in both of them, one of them, or not at all. We index the individual growth opportunity by $j \in \{A, B\}$ and denote the respective investment decision by $K_j \in \{0, 1\}$. The return on investment ($x_j^{\theta_j}$) depends on the realization of the binary random variable $\theta_j \in \{L, H\}$ which takes on the two values "high" ($H$) and "low" ($L$) with equal probability, $P(\theta_j = H) = P(\theta_j = L) = \frac{1}{2}$. For simplicity, we assume that $\theta_A$ and $\theta_B$ are independent of each other and all other random variables in the model. Moreover, we make the following three assumptions regarding the return on investment.

**Assumption 1 (Return on Growth Opportunities)** We assume that the returns on the growth opportunities satisfy the following three conditions:

(i) $x_j^L < 0 < x_j^H$ for $j \in \{A, B\}$: ex post, it is efficient to invest in project $j$ if $\theta_j = H$ and inefficient if $\theta_j = L$;

(ii) $x_j^L + x_j^H \leq 0$ for $j \in \{A, B\}$: ex ante, it is inefficient to invest in project $j$;

(iii) $x_A^H \geq x_B^H$: in the "high" state investment opportunity $A$ offers a (weakly) higher return than investment opportunity $B$.

The first assumption is necessary to make the firm’s investment problem interesting. If $x_j$ was always positive (negative), the firm would always (never) invest and there would be no role for
learning about $\theta_j$ which is the focus of this paper. The second assumption is made for tractability. It implies that the firm manager does not invest based on prior information, i.e. it requires a positive signal about $\theta_j$ to induce the manager to invest.\textsuperscript{4} Our results are robust to the alternative assumption that the ex ante NPV is positive in which case it requires a negative signal to induce the manager not to invest. The third assumption allows for an asymmetric return on the two growth opportunities. Without loss of generality, we define $A$ to be the project with the (weakly) higher return if $\theta = H$.\textsuperscript{5} In the following, we will sometimes refer to the relative return of both projects, as the degree of diversity which is formally defined as:\textsuperscript{6}

$$\kappa_x \equiv \frac{x^H_B}{x^H_A} \in (0, 1].$$

(1)

Intuitively, a higher value of $\kappa_x$ indicates that the two growth opportunities offer a more similar return on investment. In the limit $\kappa_x \rightarrow 0$, there is no diversity because it is never efficient for the firm to invest in project $B$ and there is effectively only one (potentially profitable) project.

The firm’s terminal value comprises three parts, the return on the assets-in-place and the return on both growth opportunities:

$$V = V_0 + \sum_{j \in \{A, B\}} K_j x^\theta_j.$$

(2)

The constant $V_0$ represents the return on the firm’s assets-in-place. To highlight the feedback effect from stock prices to real investment and to keep the equilibrium expressions more compact, we focus purely on the growth opportunities and set $V_0 = 0$ in the following.

Since the manager chooses $K_j$ to maximize the expected firm value, she invests ($K_j = 1$) if her conditional expectation of $x^\theta_j$ is positive and does not invest ($K_j = 0$) otherwise. It will become clear below that the information structure implies the manager either learns $\theta_j$ perfectly or not at all. Thus, the manager only invests in project $j$ when she learns that $\theta_j = H$ from the financial market or her private signal.

\textsuperscript{4}We assume throughout that the manager does not invest when she is indifferent.
\textsuperscript{5}It will become clear below that the firm never invests if $\theta = L$ and thus the actual levels of $x^L_j$ are irrelevant; it only matters that they are (sufficiently) negative.
\textsuperscript{6}It is worth noting that our diversity measure is based on payoff primitives, while Goldstein and Yang [2015] define information diversity in terms of the equilibrium mass of traders along both dimensions. We will show below that the two diversity measures are closely connected.
Next, we describe the two signals that influence the manager’s investment decision in more
detail. The first signal type is an endogenous feedback signal from the financial market based on
the stock price $P$. The second type is a private signal $\sigma_m \in \{I_m \theta_A + (1 - I_m) \theta_B, \emptyset\}$ with $I_m \in \{0, 1\}$. Thus, the manager’s choice of $I_m$ determines whether she receives an informative signal about
growth opportunity $A$ or $B$. This assumption captures the fact that in reality firm managers face
several constraints limiting their ability to acquire precise private information about all dimensions
of uncertainty.\(^7\) This assumption allows us to focus on the coordination problem between the firm
manager and informed traders with regards to their information choice. Moreover, we set the
precision of the manager’s private signal equal to $\Gamma_m$ such that $P(\sigma_m = \emptyset) = 1 - \Gamma_m$. The manager’s
information choice maximizes the firm’s expected value at $t = 0$:

$$\max_{I_m \in \{0, 1\}} E[V]$$  \hspace{1cm} (3)

where $V$ is defined in equation (2). It is important to note that the firm’s terminal value $V$
depends on the optimal real investment decisions $K_j$. As we will show below, receiving informative
signals allows the manager to invest more efficiently which, in turn, raises the firm’s expected value.
In the following, we normalize $\Gamma_m = 1$ and model the firm manager as the best-informed agent in
the economy who always receives a perfect signal about one of the two projects. At the same time,
she might be able to learn additional information about the other project from the stock price.\(^8\)

### 2.2 The Financial Market

Trading at $t = 1$ is modeled in the spirit of Kyle [1985]. The financial market consists of the
following three types of traders who trade claims to the firm’s liquidating dividend $V$ at a price $P$.
First, a unit continuum of risk-neutral informed traders, indexed by $i \in [0, 1]$. Each trader can either
buy up to one unit, sell up to one unit, or not trade at all, $s_i \in [-1, 1]$.\(^9\) Because traders do not have
price impact and are risk-neutral, they will always trade up to the limits if they decide to trade.\(^10\)

\(^7\)One possible friction could be a capacity constraint as in the literature on limited attention [see e.g. van Nieuwerburgh and Veldkamp, 2010; Kacperczyk et al., 2016, among others].

\(^8\)This normalization also allows us to turn off market feedback along the dimension the manager specializes in.

\(^9\)A potential justification for this position limit are borrowing or short-sell constraints. See also Goldstein et al. [2013] and Goldstein and Yang [2019] for feedback models with the same assumption.

\(^10\)Following the existing literature, we assume that traders do not trade if they are indifferent.
In addition to informed traders, noise traders collectively demand $z \sim U[-1, 1]$ which generates non-fundamental variation in total order flow. Lastly, a risk-neutral, competitive market maker sets the stock price based on aggregate order flow $X = \int_0^1 s_i di + z$ to break even in expectation:

$$P = \mathbb{E}[V | X].$$

(4)

Informed traders face the same learning technology as the firm manager. Each trader receives a private signal $\sigma_i \in \{I_i \theta_A + (1 - I_i) \theta_B, \emptyset\}$ with $I_i \in \{0, 1\}$. Just like the firm manager, each trader has to decide whether to acquire information about project A or B. This private signal reveals information about the fundamental with probability $\Gamma \in (0, 1]$ such that $\mathbb{P}(\sigma_i = \emptyset) = 1 - \Gamma$. Traders choose $I_i$ to maximize their expected trading profit $\Pi_i = s_i (V - P)$:

$$\max_{I_i \in \{0, 1\}} \mathbb{E}[\Pi_i].$$

(5)

It is worth noting that the expected trading profits also depend on the manager’s real investment decisions, which impact $V$. Thus, the traders’ objective is not only affected by their own information choice but also by the manager’s. We will elaborate more on this strategic interaction below. It follows from our previous assumption $\Gamma_m = 1$ that the traders’ information capacity $\Gamma$ can be interpreted as their degree of sophistication relative to the manager’s.

2.3 Equilibrium

Our equilibrium concept is Perfect Bayesian Equilibrium ("PBE"). We conjecture, and verify in Proposition 3, that there might be circumstances without a pure-strategy information equilibrium. In these cases, we let the firm manager and informed traders choose mixing probabilities $\{\omega_A, \omega_B\}$ and $\{q_{i,A}, q_{i,B}\}$, respectively. Moreover, we restrict attention to symmetric information acquisition decisions for traders and assume that their private signals are uncorrelated among each other. Thus for a given mixing probability $q_{i,A} = q_A$ and $q_{i,B} = q_B$ the mass of traders with a perfect signal about $\theta_A$ and $\theta_B$ is equal to $q_A \Gamma$ and $q_B \Gamma$, respectively by the law of large numbers. Furthermore, we also conjecture, and verify in Proposition 3, that $q_A \geq q_B$ which is intuitive because we assume project A to yield the weakly higher ex post return. Of course, a pure-strategy information equilibrium
for traders corresponds to \( q_A = 1 \) (or \( q_A = 0 \)). The firm manager can also randomize between the two signals by choosing the mixing probability \( \omega_A \) associated with \( I_m = 1 \). It follows from our assumption that \( \Gamma_m = 1 \) that the manager observes \( \theta_A \) with probability \( \omega_A \) and \( \theta_B \) with probability \( \omega_B = 1 - \omega_A \).

**Definition 1 (Perfect Bayesian Equilibrium)** A PBE consists of the following two sub-equilibria.

1. **Trading and investment equilibrium at** \( t = 1 \):
   - informed traders choose their asset demands to maximize expected trading profits;
   - the market maker sets the price to break even in expectation;
   - the firm manager chooses capital investments to maximize the expected firm value.

2. **Information acquisition equilibrium at** \( t = 0 \):
   - traders acquire information to maximize expected profits anticipating the equilibrium at \( t = 1 \);
   - the manager acquires information to maximize the expected firm value anticipating the equilibrium at \( t = 1 \).

We assume that all agents have rational expectations in that each player’s belief about the other players’ strategies is correct in equilibrium.

3 **Model Solution and Implications**

   In this section we characterize the equilibrium in the main model and discuss its implications. More specifically, we solve the model backwards and start with the financial market and investment equilibrium at \( t = 1 \) in Section 3.1. In Section 3.2, we derive the information acquisition equilibrium at \( t = 0 \). Section 3.3 discusses the model’s efficiency implications.

3.1 **Trading and Investment Equilibrium**

   As a first step, we take all agents’ information choices at \( t = 0 \) as given. Therefore, each trader acquires information about the two fundamental shocks with probabilities \( q_A \) and \( q_B \), respectively.\(^{11}\)

\(^{11}\)As mentioned earlier, we conjecture, and verify in Proposition 3, that \( q_A \geq \frac{1}{2} \) in equilibrium.
Similarly, the firm manager’s information choice is described by the mixing probabilities $\omega_A$ and $\omega_B$. Based on these choices, each trader has to decide whether to trade and, if yes, whether to buy or sell the asset. Furthermore, the market maker sets the equilibrium stock price based on the observed order flow and the firm manager decides whether to invest in the growth opportunities.

**Proposition 1 (Trading and Investment Equilibrium)** Given information choices by traders ($q_A$) and the manager ($\omega_A$), there is a trading and investment equilibrium in which:

1. Each trader buys the firm’s stock if $\sigma_i = H$, sells if $\sigma_i = L$, and does not trade if $\sigma_i = \emptyset$:

   $$s_i = \begin{cases} 
   +1 & \text{if } \sigma_i = H \\
   -1 & \text{if } \sigma_i = L \\
   0 & \text{if } \sigma_i = \emptyset 
   \end{cases}$$

   which leads to the following total order flow:

   $$X = \begin{cases} 
   \Gamma + z & \text{if } \theta_A = H, \theta_B = H \\
   (2q_A - 1) \Gamma + z & \text{if } \theta_A = H, \theta_B = L \\
   (1 - 2q_A) \Gamma + z & \text{if } \theta_A = L, \theta_B = H \\
   -\Gamma + z & \text{if } \theta_A = L, \theta_B = L 
   \end{cases}$$

2. The firm’s stock price satisfies:

   $$P = \begin{cases} 
   p_{HH}^H = x_A^H + x_B^H & \text{if } X \in (1 + (2q_A - 1) \Gamma, 1 + \Gamma) \\
   p_H = x_A^H + \frac{1}{2} \omega_B x_B^H & \text{if } X \in (1 - (2q_A - 1) \Gamma, 1 - (2q_A - 1) \Gamma) \\
   p_{\emptyset,H}^H = \frac{1}{3} \left( (1 + \omega_A)x_A^H + (1 + \omega_B)x_B^H \right) & \text{if } X \in (1 - \Gamma, 1 - (2q_A - 1)\Gamma) \\
   p_{\emptyset,M}^H = \frac{1}{2} \left( \omega_A x_A^H + \omega_B x_B^H \right) & \text{if } X \in (-1 + \Gamma, 1 - \Gamma) \\
   p_{\emptyset,L}^H = \frac{1}{3} \left( \omega_A x_A^H + \omega_B x_B^H \right) & \text{if } X \in (-1 - (2q_A - 1)\Gamma, -1 + \Gamma) \\
   p_L = \frac{1}{2} \omega_B x_B^H & \text{if } X \in (-1 - (2q_A - 1)\Gamma, -1 + (2q_A - 1)\Gamma) \\
   p_{LL}^H = 0 & \text{if } X \in (-1 - \Gamma, -1 - (2q_A - 1)\Gamma) 
   \end{cases}$$
3. The firm’s investment decision satisfies:

\[ K_A = \begin{cases} 
1 & \text{if } (\sigma_m = \theta_A = H) \lor (P \in \{p^{H_H}, p^{H_H}\}) \lor (\sigma_m = \theta_B = L \land P = p^{\emptyset, H}) \\
0 & \text{otherwise}
\end{cases} \]

and

\[ K_B = \begin{cases} 
1 & \text{if } (\sigma_m = \theta_B = H) \lor (P = p^{H_H}) \lor (\sigma_m = \theta_A = L \land P = p^{\emptyset, H}) \\
0 & \text{otherwise}
\end{cases} \]

In these expressions we used \( \omega_B = 1 - \omega_A \). Moreover, we provide the required conditions on \( q_A \) and \( \omega_A \) such that the traders’ expected profits are positive in the Appendix.

**Proof:** See Appendix A.2.1.

Proposition 1 shows that each trader optimally buys (sells) on positive (negative) private information about the firm’s fundamental in anticipation of a higher (lower) payoff \( V \).\(^{12}\) It follows that total order flow \( X \) depends on the informed traders’ private signals and therefore the fundamentals \( \theta_j \). In a mixed-strategy information equilibrium, the resulting aggregate order flow will be a function of \( \theta_A \) and \( \theta_B \) because a fraction \( q_A \Gamma \) receives a perfect signal about \( \theta_A \) and a fraction \( q_B \Gamma \) receives a perfect signal about \( \theta_B \). In this case, a particularly high order flow indicates that both shocks must be equal to \( \theta_j = H \) because it must be a result of buy orders from both types of informed investors. The same intuition applies to a particularly negative order flow such that both shocks must be equal to \( \theta_j = L \). It should be noted that there is an inherent asymmetry between the two shocks \( \theta_A \) and \( \theta_B \). Due to the assumption that project \( A \) offers a weakly higher ex post return, we will show below in Proposition 3 that there are always weakly more \( A \)-informed traders in a mixed-strategy equilibrium \( (q_A \geq q_B) \) such that the stock price reveals weakly more information about \( \theta_A \). More precisely, slightly above or below-average values of \( X \) already reveal \( \theta_A = H \) or \( \theta_A = L \) to the market maker. However, it requires more extreme values for \( X \) to reveal the value of \( \theta_B \) together with that of \( \theta_A \).

There are two interesting corner solutions with regards to the traders’ information choice. First,\(^{13}\)

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\(^{12}\)Note that this strategy is only profitable for certain values of \( q_A \) and \( \omega_A \). However, we show in Proposition 3 that these conditions are always met at the equilibrium choices \( q_A^* \) and \( \omega_A^* \) such that the equilibrium above holds for all permissible parameter values.
if \( q_A = q_B = \frac{1}{2} \), traders randomize evenly between both shocks. In this case, the market maker cannot disentangle \((\theta_A = H, \theta_B = L)\) from \((\theta_A = L, \theta_B = H)\) because both combinations lead to the same total order flow \( X = z \sim U[-1, 1] \). As a result, he learns that both shocks are "high" if \( X > 1 \) and that both shocks are "low" if \( X < -1 \). If \( X \in (1 - \Gamma, 1) \) or \( X \in (-1, -1 + \Gamma) \), the market maker knows that the manager might invest in project \( j \) if her signal about the other project is low. It follows that the equilibrium stock price only takes on five distinct values. If the market maker learns that both shocks are "high," he sets the price equal to \( p^{HH} \), if he learns that both shocks are "low," he sets the price equal to \( p^{LL} \), if he cannot infer any information from \( X \), the price is set equal to \( p^{\emptyset,M} \). In the other two scenarios, he sets the price equal to \( p^{\emptyset,H} \) and \( p^{\emptyset,L} \), respectively.

The second interesting case is the pure-strategy information equilibrium, \( q_A = 1 \). In this case, all traders specialize in \( \theta_A \) and total order flow cannot reveal any information about \( \theta_B \). As a result, the stock price takes on the value \( p^H \) if the market maker learns that \( \theta_A = H, p^L \) if he learns that \( \theta_A = L, \) and \( p^{\emptyset,M} \) otherwise.

More generally, we can see from Proposition 1 that the equilibrium stock price \( P \) is a step function of total order flow \( X \). The two most extreme values \( p^{HH} \) and \( p^{LL} \) reveal that the market maker has learnt \( \theta_A = \theta_B = H \) and \( \theta_A = \theta_B = L \), while the two less extreme values \( p^H \) and \( p^L \) reveal that he has learnt \( \theta_A = H \) and \( \theta_A = L \). The intermediate value \( p^{\emptyset,M} \) is uninformative. A price slightly above average (\( p^{\emptyset,H} \)) or below average (\( p^{\emptyset,L} \)), reveals very little information about \( \theta_A \) and \( \theta_B \) per se, but might reveal the value of \( \theta_j \) to the firm manager once combined with her private signal.

Put together, we can see that the firm manager either learns the true value of \( \theta_j \) from her private signal or the stock price. Given that she invests benevolently, the manager invests in project \( j \) if she learns \( \theta_j = H \) and does not invest otherwise. Therefore, she invests in growth opportunity \( A \) if the price is equal to \( p^H \) or \( p^{HH} \) while she invests in \( B \) if the price is equal to \( p^{HH} \). Of course, the decision to invest in either growth opportunity can also be triggered by a positive realization of the manager's private signal \( \sigma_m \). In a pure-strategy equilibrium this signal either reveals \( \theta_A \) or \( \theta_B \) perfectly. In a mixed-strategy equilibrium, it reveals \( \theta_A \) with probability \( \omega_A \) and \( \theta_B \) otherwise.
Most interestingly, the manager also invests in $\theta_j$ if her private signal about the other shock is low and the price is equal to $p^{0,H}$.

### 3.2 Information Choice Equilibrium

Next, we analyze the information acquisition decisions at the initial date $t = 0$ and compute the firm manager’s and traders’ expected utilities. Starting with the former, we use the manager’s optimal investment policy in Proposition 1 and the definition of $V$ in equation (2) to show that the expected firm value is given by:

$$\mathbb{E}[V] = \left( \omega_A + (1 - \omega_A)q_A \Gamma \right) x_A^H + \frac{(1 - \omega_A + \omega_A(1 - q_A) \Gamma) x_B^H}{2}. \quad (6)$$

Next, we differentiate this expression with respect to $\omega_A$ to obtain the manager’s marginal benefit of acquiring information about $\theta_A$:

$$\frac{\partial \mathbb{E}[V]}{\partial \omega_A} = \frac{(1 - q_A \Gamma) x_A^H - (1 - (1 - q_A) \Gamma) x_B^H}{2}. \quad (7)$$

We can use this expression to solve for the manager’s best response to the traders’ collective information choice $q_A$:

$$\omega_A^* = \begin{cases} 
1 & \text{if } q_A < \frac{1-(1-\Gamma)\kappa_x}{(1+\kappa_x)\Gamma} \\
\epsilon \in [0, 1] & \text{if } q_A = \frac{1-(1-\Gamma)\kappa_x}{(1+\kappa_x)\Gamma} \\
0 & \text{if } q_A > \frac{1-(1-\Gamma)\kappa_x}{(1+\kappa_x)\Gamma}.
\end{cases} \quad (7)$$

Because the manager’s objective function is linear in her mixing probability $\omega_A$, we either get a corner solution or the manager is indifferent between all values for the mixing probability. In particular, equation (7) shows that if $q_A$ is below (above) the threshold value $\frac{1-(1-\Gamma)\kappa_x}{(1+\kappa_x)\Gamma}$, the manager chooses to acquire information about $\theta_A$ ($\theta_B$) all the time. If $q_A$ is equal to this value, the manager is indifferent between any value for $\omega_A$ and willing to pursue a mixed strategy.

Next, we compute the expected trading profits for trader $i$ to find the best response of each individual trader to the manager’s information choice. Under the most general assumption that the manager chooses a mixing probability $q_{i,A}$, the trader receives a perfect signal about $\theta_A$ with probability $q_{i,A} \Gamma$ and a perfect signal about $\theta_B$ with probability $(1 - q_{i,A}) \Gamma$. Hence, we can write
the trader’s objective function as

$$E[\Pi_i] = q_{i,A} \mathbb{E}[\Pi_i | \sigma_i = \theta_A] + (1 - q_{i,A}) \mathbb{E}[\Pi_i | \sigma_i = \theta_B]$$

(8)

because, according to Proposition 1, traders do not trade ($s_i = 0$) if they receive an uninformative signal ($\sigma_i = \emptyset$). We can differentiate this expression with respect to $q_{i,A}$ to get

$$\frac{\partial E[\Pi_i]}{\partial q_{i,A}} = \Gamma (\mathbb{E}[\Pi_i | \sigma_i = \theta_A] - \mathbb{E}[\Pi_i | \sigma_i = \theta_B]).$$

We can see that at an interior optimum each trader is indifferent between specializing in either project, i.e. $\mathbb{E}[\Pi_i | \sigma_i = \theta_A] = \mathbb{E}[\Pi_i | \sigma_i = \theta_B]$. Of course, if $\mathbb{E}[\Pi_i | \sigma_i = \theta_A] > \mathbb{E}[\Pi_i | \sigma_i = \theta_B]$ it is optimal to specialize in $\theta_A$, i.e. $q_{i,A} = 1$, and vice versa if $\mathbb{E}[\Pi_i | \sigma_i = \theta_A] < \mathbb{E}[\Pi_i | \sigma_i = \theta_B]$.

Next, we can plug in trader $i$’s realized profit $\Pi_i = s_i(V - P)$ together with the results in Proposition 1 to see that the individual trader’s expected profit conditional on $\sigma_i$ only depends on the collective information choice of all traders ($q_A$), that of the manager ($\omega_A$), and the model parameters $\kappa_x$ and $\Gamma$. Lastly, we set $q_{i,A}^* = q_A^*$ and derive the traders’ best-response function to the manager’s information choice:

$$q_A^* = \begin{cases} 1 & \text{if } \omega_A > \frac{\kappa_x}{1 + \kappa_x - \Gamma} \\ \frac{\Gamma - \kappa_x + \omega_A(1 + (1 - \Gamma)\kappa_x)}{1 + \omega_A(1 - \kappa_x)} & \text{if } \frac{\kappa_x - \Gamma}{1 + \omega_A(1 - \kappa_x)} \leq \omega_A \leq \frac{\kappa_x}{1 + \kappa_x - \Gamma} \\ 0 & \text{if } \omega_A < \frac{\kappa_x - \Gamma}{1 + \omega_A(1 - \kappa_x)}. \end{cases}$$

(9)

Thus, the traders’ optimal mixing probability depends on the manager’s mixing probability. For intermediate values of $\omega_A$, the mixing probability is equal to $\frac{\Gamma - \kappa_x + \omega_A(1 + (1 - \Gamma)\kappa_x)}{1 + \omega_A(1 - \kappa_x)}$. Since this expression is increasing in $\omega_A$, we find that informed traders choose a pure strategy and specialize in $\theta_A$ if $\omega_A$ is sufficiently high. If $\omega_A$ is sufficiently small, informed traders move to the other extreme and choose $q_A = 0$. We will show below that this scenario never occurs in equilibrium. Overall, we can see from the expressions for $q_A^*$ and $\omega_A^*$ that the manager’s and the market’s information choice are interdependent. The following proposition formalizes this interdependence.

**Proposition 2 (Substitutability vs. Complementarity)** From the firm’s perspective, market informa-
tion and the manager’s private information are strategic substitutes:

\[ \frac{\partial \omega^*_A}{\partial q^*_A} \leq 0. \]

From the traders’ perspective, market information and the manager’s private information are strategic complements:

\[ \frac{\partial q^*_A}{\partial \omega^*_A} \geq 0. \]

**Proof:** See Appendix A.2.2.

Proposition 2 shows a key driver behind our main results. There is a fundamental tension between the firm manager and the market with regards to their incentives to acquire information. On the one hand, the firm manager has an incentive to diversify her information sources about the two investment opportunities. Thus, if the stock price is particularly informative about \( \theta_A \) because \( q_A \) is high, she would rather pay more attention to \( \theta_B \) and vice versa. This result is intuitive because the manager wants to avoid “overlap” in the information conveyed by the price signal and her private signal. For instance, if the price signal already reveals \( \theta_A = H \) (e.g. because \( P = p^H \)) receiving the same information from the private signal does not add any value. On the other hand, traders have an incentive to focus on the same shock as the manager because it allows them to predict the payoff, that depends on the manager’s investment choice, more precisely. In the extreme case in which the manager solely focuses on \( \theta_A \), traders don’t have an incentive to acquire information about \( \theta_B \) at all. If they did, the manager would only invest in project \( B \) in response to price signals \( P = p^{HH} \) or \( P = p^{\emptyset,H} \) (depending on her private signal about \( \theta_A \)). In the first case, the information is fully priced-in because the market maker is also able to infer \( \theta_B \) from total order flow whenever the manager invests. In the second case, private information about \( \theta_B \) does not help to predict the manager’s investment decision because it also depends on the realization of \( \theta_A \).

Next, we can combine the manager’s and traders’ demand for information in (7) and (9) to solve for the equilibrium values of \( q_A \) and \( \omega^*_A \).

**Proposition 3 (Information Choice Equilibrium)** There are two mutually exclusive information choice equilibria depending on the traders' signal precision \( \Gamma \) and the degree of diversity \( \kappa_x = \frac{x^H}{x^A} \in (0, 1] \).
1. If $\Gamma \leq 1 - \kappa$, there is a pure-strategy information acquisition equilibrium. The manager and each trader acquire information solely about $\theta$: 

$$ \omega^*_A = q^*_A = 1. $$

2. If $\Gamma > 1 - \kappa$, there is a mixed-strategy information acquisition equilibrium with:

$$ \omega^*_A = \frac{1 - \Gamma + \kappa^2}{2\kappa(2 - \Gamma)} $$

$$ q^*_A = \frac{1 - (1 - \Gamma)\kappa}{(1 + \kappa)\Gamma}. $$

with $\frac{\partial \omega^*_A}{\partial \Gamma} \leq 0$, $\frac{\partial \omega^*_A}{\partial \kappa} > 0$ if $\Gamma > 1 - \kappa^2$ and $\leq 0$ otherwise; $\frac{\partial q^*_A}{\partial \Gamma} \leq 0$ and $\frac{\partial q^*_A}{\partial \kappa} \leq 0$; $\omega^*_A \in [0, \frac{1}{2}]$ and $q^*_A \in [\frac{1}{2}, 1]$.

**Proof:** See Appendix A.2.3.

Proposition 3 gives the equilibrium values of the mixing probabilities $\omega^*_A$ and $q^*_A$. We can see that there are two different regions. First, if the traders’ precision capacity $\Gamma$ is sufficiently low, all agents focus their attention on shock $A$, i.e. $\omega^*_A = q^*_A = 1$. This result is intuitive: in anticipation of the market’s low precision capacity, the manager’s incentive to shift her attention towards shock $B$ is reduced because she does not expect the equilibrium price to be very informative about shock $A$. Therefore, it is optimal for the manager to specialize in shock $A$ which, in turn, incentivizes all traders to mimic the manager’s choice and specialize in shock $A$, too. Second, if the traders’ information capacity ($\Gamma$) is above a certain threshold, the manager and each trader randomize between shock $A$ and $B$. In this case, the mixing probabilities depend on the degree of diversity in the two projects $\kappa$ and the traders’ signal precision $\Gamma$.

The comparative statics of the two mixing probabilities can be most easily seen in Figure 2. The traders’ mixing probability for project $A$ is monotonically decreasing in both $\kappa$ and $\Gamma$. As project $B$ becomes relatively more profitable, traders start shifting more attention towards this project and $q^*_A$ converges towards $\frac{1}{2}$ from above. Similarly a decrease in signal precision $\Gamma$ makes it less likely that an individual traders’ signal is revealed to the market maker. As a result, it is less costly to pay attention to the relatively more profitable shock $A$. One can also see that our initial conjecture
holds and \( q_A^* \geq \frac{1}{2} \) for all permissible values of \( \Gamma \) and \( \kappa_x \). The manager’s mixing probability looks quite different and is rather flat in both dimensions. We can show analytically that \( \omega_A^* \) is always between 0 and \( \frac{1}{2} \) in the feedback region \( \Gamma > 1 - \kappa_x \) (blue area) and equal to unity in the non-feedback region \( \Gamma \leq 1 - \kappa_x \) (red area).

![Figure 2: Equilibrium values for \( \omega_A^* \) (left plot) and \( q_A^* \) (right plot) as a function of \( \kappa_x = \frac{\kappa}{xy} \) and \( \Gamma \).](image)

As mentioned above, Goldstein and Yang [2015] use a slightly different concept of information diversity (\( \Delta \)) that relies on the equilibrium mass of \( A \)-informed and \( B \)-informed traders. Using our notation, this alternative measure is given by:

\[
\Delta \equiv 1 - \frac{\left(q_A^* - q_B^*\right)}{\left(q_A^* + q_B^*\right)} \Gamma = \frac{2(1 - q_A^*)}{\Gamma} \in [0, 1].
\] (10)

Using the results in Proposition 3, it is straightforward to show that this measure is increasing in \( \kappa_x \), i.e. \( \frac{\partial \Delta}{\partial \kappa_x} \geq 0 \). As a result, both measures of diversity are closely connected.

### 3.3 Efficiency Implications

Next, we analyze the efficiency implications of the manager’s and the traders’ strategic information acquisition decisions. In particular, we focus on two widely used efficiency measures, market efficiency and real efficiency. Both measures are formally defined next.

**Definition 2 (Efficiency)** We define the following two measures of efficiency.

1. Market efficiency is defined as the negative ratio of the (expected) conditional and unconditional payoff
2. Real efficiency is defined as the ex ante expected firm value:

\[ RE \equiv \mathbb{E}[V]. \]

First, our definition of market efficiency (\( ME \)) is the informational content of the price, i.e. the negative payoff variance conditioned on the stock price \( P \), scaled by the unconditional variance to obtain a relative measure. Note that in our setting the conditional variance is a random variable that depends on the specific price realization. For example, \( \text{Var}(V|P = p^{HH}) = 0 \) because \( p^{HH} \) perfectly reveals \( \theta_A \) and \( \theta_B \) (and thus \( V \)) while \( \text{Var}(V|P = p^{0,M}) = \text{Var}(V) \) because \( p^{0,M} \) does not reveal any information. This measure of market efficiency has been often used in the existing literature as a proxy for the informational content of the stock price (see e.g. Peress [2010], Ozsoylev and Walden [2011], and Edmans et al. [2016]).

In our setting, the expected variance ratio is also proportional to the price-payoff correlation. It is straightforward to show that \( ME = \text{Corr}(V,P)^2 - 1 \). Our measure of market efficiency is maximized at \( ME = 0 \), if the price always reveals the future payoff perfectly, and minimized at \( ME = -1 \), if observing the price does not add any information and \( P = p^{0,M} \) with probability one. Second, we define real efficiency (\( RE \)) as the ex ante expectation of the firm’s realized long-term value \( V \). It measures the efficiency of the firm’s allocation of capital and is maximized at \( \frac{1}{2} \left( x_H^A + x_H^B \right) \) if the firm manager always invests in project \( j \) when \( \theta_j = H \) (which happens with probability \( \frac{1}{2} \)).

**Proposition 4 (Efficiency Measures)** In the main model the two efficiency measures are given by:

1. In recent empirical work, Davila and Parlatore [2019a] and Davila and Parlatore [2019b] use price volatility and regression R-squareds to identify price efficiency.
2. Note that \( -\frac{\mathbb{E}[\text{Var}(V|P)]}{\text{Var}(V)} = \frac{\text{Var}(V|P)}{\text{Var}(V)} - 1 = \frac{\text{Var}(P)}{\text{Var}(V)} - 1 \). Moreover, it holds that \( \text{Cov}(V,P) = \text{Var}(P) \) because \( \mathbb{E}[VP] = \mathbb{E}[V]\mathbb{E}[P] = \mathbb{E}[\mathbb{E}[V|P]]\mathbb{E}[P] = \mathbb{E}[P] - \mathbb{E}[P]^2 \) such that \( \text{Corr}(V,P) = \sqrt{\frac{\text{Var}(P)}{\text{Var}(V)}} \). Note that the expected firm value can be written as \( \mathbb{E}[V] = \frac{1}{2} \sum_{j \in \{A,B\}} \mathbb{P}(K_j = 1|\theta_j = H) x_j^{Hj} \) because the manager never invests if \( \theta_j = L \). Hence, real efficiency captures the extent to which the firm invests in the growth opportunities when their ex post return is in fact positive.
1. Market efficiency:

\[
ME^* = \begin{cases} 
2\Gamma^3 x_A^2 + 3\Gamma^3 x_2^2 - \Gamma^3 - 35\Gamma^2 x_A^2 - 5\Gamma^2 x_2^2 - 11\Gamma^2 x_1 + 3\Gamma^2 + 28\Gamma x_A^2 + 4\Gamma x_2^2 + 13\Gamma x_1 - 3\Gamma - 59 x_A^2 - x_2^2 - 5 x_1 + 1 \\
12(2-\Gamma)^2 \frac{\kappa_A^2}{\kappa_T^2 + 1} \left( -\Gamma x_A^2 + 4\Gamma x_2^2 + x_1^2 + 2 x_2^2 - 2 x_1 + 1 \right) \\
\Gamma - 1 
\end{cases}
\]

if \( \Gamma > 1 - \kappa_x \)

\[
\frac{\partial ME^*}{\partial \kappa_x} \leq 0 \quad \text{and} \quad \frac{\partial ME^*}{\partial \Gamma} > 0; \quad \text{moreover,} \quad ME^* \in (-1, 0).
\]

2. Real efficiency:

\[
RE^* = \begin{cases} 
\frac{1+\kappa_x (\Gamma + \kappa_x)}{2(1+\kappa_x)} x_A^H \quad \text{if} \quad \Gamma > 1 - \kappa_x \\
\frac{1}{2} x_A^H \quad \text{if} \quad \Gamma \leq 1 - \kappa_x 
\end{cases}
\]

with \( \frac{\partial RE^*}{\partial \kappa_x} \geq 0 \quad \text{and} \quad \frac{\partial RE^*}{\partial \Gamma} \geq 0; \quad \text{moreover,} \quad RE^* \in \left[ \frac{1}{2} x_A^H, \frac{3}{4} x_A^H \right].
\]

Again, \( \kappa_x = \frac{x_B^H}{x_A^H} \in (0, 1] \) denotes the degree of diversity in the two projects.

**Proof:** See Appendix A.2.4.

Proposition 4 provides analytic expressions for our two efficiency measures. As before, these expressions depend on the traders’ signal precision \( \Gamma \) and the degree of project diversity \( \kappa_x \). If \( \Gamma \leq 1 - \kappa_x \), the manager and traders specialize in \( \theta_A \) and there is no market feedback. In this case, real efficiency is solely a function of the return on project \( A, x_A^H \), and market efficiency is solely a function of the traders’ information capacity \( \Gamma \). If \( \Gamma > 1 - \kappa_x \), the manager randomizes between an informative signal about \( \theta_A \) and an informative signal about \( \theta_B \). As a result, she is able to learn additional information about the shock, that is not perfectly revealed by her private signal, from the stock price. In this scenario, real efficiency depends positively on the precision of the informed traders’ signal because this precision determines the informational content of the price. Furthermore, as expected, real efficiency also increases in the high-state payoff of both projects. With price feedback, market efficiency depends on the degree of diversity \( \kappa_x \) and the following corollary describes this dependence in more detail.

**Corollary 1 (Impact of Information Diversity)** In the main model, information diversity \( \kappa_x = \frac{x_B^H}{x_A^H} \) affects the two efficiency measures in the following way. For a given \( \Gamma \in (0, 1] \):
1. **Real efficiency is maximized at** $\kappa_x = 1$.

2. **Market efficiency is maximized at** $\kappa_x \downarrow 1 - \Gamma$ for $\Gamma \leq \widehat{\Gamma}$ and at $\kappa_x = 1$ otherwise.

*The threshold $\widehat{\Gamma}$ is given in the Appendix.*

**Proof:** See Appendix A.2.5.

Quite interestingly, the two efficiency measures respond differently to changes in the degree of information diversity $\kappa_x$. More specifically, real efficiency is strictly increasing in $\kappa_x$ such that the firm’s expected value is maximized if the relative payoff of the two projects is equal to unity and minimized if it is equal to zero. Market efficiency, on the other hand, is non-monotone in $\kappa_x$. Most strikingly, market efficiency can be *maximized* when real efficiency is *minimized*, i.e. in when $\kappa_x$ approaches $1 - \Gamma$ from above.

Figure 3 plots our two efficiency measures against the traders’ signal precision $\Gamma$ and the degree of information diversity $\kappa_x$. First, the pure-strategy, no-feedback case corresponds to the triangle in the front left of both plots ($\Gamma \leq 1 - \kappa_x$). In this region, market efficiency increases linearly in $\Gamma$ and does not depend on $\kappa_x$. Real efficiency only depends on $x_A^H$, which is set to unity in this plot, because the firm manager invests in project $A$ only based on her private signal. Second, the mixed-strategy, feedback case corresponds to the triangle in the back right of both plots ($\Gamma > 1 - \kappa_x$). We can see that the two efficiency measures differ quite dramatically in this region. On the one hand, market efficiency is maximized at $\Gamma \rightarrow 1$ and $\kappa_x \rightarrow 0$. In this case, the firm’s payoff only depends on shock $A$ such that traders only acquire information about $\theta_A$. If, in addition, $\Gamma = 1$ the stock price is fully information about this shock and the firm manager can extract the informed traders’ information perfectly. As a result, price efficiency is maximized and $ME^* = 0$. On the other hand, real efficiency is maximized at $\Gamma \rightarrow 1$ and $\kappa_x \rightarrow 1$. In this case, traders randomize between both shocks and it is more difficult for the manager to infer information from the price which is reflected in the reduced level of market efficiency. At the same time, however, the manager is able to learn about projects $A$ and $B$ from the stock price (in addition to her private information) which increases the overall amount of *useful* information for the manager and hence increases the ex ante firm value.
Figure 3: Equilibrium values for market efficiency (left plot) and real efficiency (right plot) as a function of $\kappa \frac{H}{x}$ and $\Gamma$. We set $x_H = 1$ for the right plot.

4 Extension: A Dismissive Manager

In this section, we consider an extension of the main model in which the firm’s manager is skeptical of the precision of the traders’ private signal. More specifically, we will consider a dismissive manager who systematically underestimates this precision and believes $P(\sigma_i = \emptyset) = 1 - \lambda \Gamma$ instead of $1 - \Gamma$. The constant $\lambda \in (0, 1)$ measures the extent of the manager’s dismissiveness and this extension simplifies to the main model if $\lambda = 1$. To keep the expressions more tractable, we will set the traders’ true capacity equal to unity in this section ($\Gamma = 1$).

Our main goal is to show how the manager’s dismissiveness affects equilibrium information choices and efficiency. It is important to note that all traders and the market maker are aware of the manager’s dismissiveness. Vice versa, the manager believes that $\lambda \Gamma$ is the true capacity and that all other agents act under this belief.

From the manager’s perspective, the expected firm value is given by:

$$\tilde{E}[V] = \frac{1}{2} (\lambda \tilde{q_A}(1 - \kappa_x \tilde{\omega}_A - \tilde{\omega}_A) - \kappa_x (1 - \lambda) \tilde{\omega}_A + \kappa_x + \tilde{\omega}_A) \times_H,$$

where $\tilde{E}[-]$ denotes the manager’s distorted expectation. It is important to note that the manager falsely believes the traders’ information choice is equal to $\tilde{q_A}$ instead of $\tilde{q_A}$ because she falsely believes that traders use $\tilde{\Gamma}$ instead of $\Gamma$. The manager’s marginal benefit of acquiring a private
signal about project $A$ is given by:

$$\frac{\partial \mathbb{E}[V]}{\partial \tilde{\omega}_A} = \frac{1}{2} (1 - \kappa_x (1 - \lambda) - (1 + \kappa_x) \lambda \tilde{q}_A) x_A^H.$$  \hspace{1cm} (12)

Traders know that their information capacity is equal to unity and that the manager underestimates it. As before, each trader’s marginal benefit from acquiring information about project $A$ is given by:

$$\frac{\partial \mathbb{E}[\Pi_i]}{\partial \tilde{q}_{i,A}} = \mathbb{E}[\Pi_i]_{\sigma_i = \theta_A} - \mathbb{E}[\Pi_i]_{\sigma_i = \theta_B},$$  \hspace{1cm} (13)

where the expected trading profits depend on $\tilde{q}_A$ and $\tilde{\omega}_A$.

To find the information choice equilibrium in this extended setting, we first solve for the manager’s choice by finding the fixed-point between $\{\tilde{\omega}_A, \tilde{q}_A\}$. Then, we use the optimal $\tilde{\omega}_A$ and plug it into the traders’ best-response function to find $\tilde{q}_A$.

**Proposition 5 (Information Choice Equilibrium with a Dismissive Manager)**  In the extended model with a dismissive manager, there are two mutually exclusive information choice equilibria.

1. If $\lambda > 1 - \kappa_x$, there is a mixed strategy information acquisition equilibrium with:

   $$\tilde{\omega}_A = \frac{1 - \lambda + \kappa_x^2}{2\kappa_x (2 - \lambda)}$$
   $$\tilde{q}_A = \frac{\kappa_x (2\kappa_x \lambda + 4 - 3\kappa_x - 2\lambda) + 1 - \lambda}{(\kappa_x + 1)(2 - \kappa_x)\kappa_x + 1 - \lambda}.$$

2. If $\lambda \leq 1 - \kappa_x$, there is a pure-strategy information acquisition equilibrium with:

   $$\tilde{\omega}_A = 1$$
   $$\tilde{q}_A = 1.$$

**Proof:** See Appendix A.2.6.

Proposition 5 describes the unique information choice equilibrium. As in the main model, there are two mutually exclusive equilibria. First, if the manager heavily underestimates the traders’ information capacity ($\lambda \leq 1 - \kappa_x$), she focuses on project $A$ and chooses $\tilde{\omega}_A = 1$. The traders’ best response is to also focus solely on this project such that $\tilde{q}_A = 1$. Second, if the manager has more
trust in the informational content of the price ($\lambda > 1 - \kappa x$), there is an interior optimum and all agents randomize between private signals about both projects.

The manager’s dismissiveness encourages her to put more weight on project $A$, $\frac{\partial q_A}{\partial \lambda} < 0$, and encourages the traders to follow the manager’s choice, $\frac{\partial q_A}{\partial \lambda} < 0$. Consequently, a lower $\lambda$ renders the stock price more informative about the more profitable project which might benefit the manager’s investment decision and thus real efficiency.

**Proposition 6 (Optimal Degree of Dismissiveness)** In the extended model with a dismissive manager, the optimal degree of dismissiveness is given by:

$$\lambda^* = \frac{1 - \kappa x (2\sqrt{2} - \kappa x + \kappa x + 2\sqrt{2} - \kappa x - 6)}{4\kappa x + 1}$$

with $\lambda^* \in (1 - \kappa x, 1)$.

**Proof:** See Appendix A.2.7.

Proposition 6 shows that real efficiency is maximized at an interior $\lambda$. Thus, the firm is always better-off hiring a dismissive manager that is neither fully dismissive ($\lambda = 0$) nor fully rational ($\lambda = 1$). Intuitively, a moderately dismissive manager encourage the traders to produce more information about the more profitable project $A$. As a result, the stock price becomes a more valuable signal for the manager and investment efficiency is increased. Figure 4 plots real efficiency against $\lambda$ and shows the optimal degree of dismissiveness.

![Figure 4](image-url)

*Figure 4:* Real efficiency as a function of $\lambda$ in the extended model with a dismissive manager (left plot) and the optimal degree of dismissiveness as a function of $\kappa x$ (right plot). For the left plot: Solid line: $\kappa x = \frac{1}{4}$; Dotted line: $\kappa x = \frac{1}{2}$; Dashed line: $\kappa x = \frac{3}{4}$. We set $x^H_A = 1$ for the left plot.
5 Conclusion

We consider a model in which a real-decision maker (the firm manager) and traders in a financial market are allowed to collect information simultaneously. The resulting information acquisition equilibrium highlights an inherent coordination problem: while the manager wants traders to acquire information that differs from her own choice, traders want to collect the same information as the manager. We show that this tension leads to a mismatch between market efficiency and real efficiency.
References


## A Appendix

### A.1 Notation Summary

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A.2 Proofs

A.2.1 Proof of Proposition 1

To prove Proposition 1, we proceed as follows. We first take the traders’ and the manager’s information choice as given. It follows that a mass \(q_A \Gamma\) receives an informative signal about \(\theta_A\) and a mass \((1 - q_A) \Gamma\) receives an informative signal about \(\theta_B\). Moreover, we conjecture that trader \(i\)’s optimal trading policy is to buy if \(\sigma_i = H\), sell if \(\sigma_i = L\), and not to trade if \(\sigma_i = \emptyset\) and that the manager follows the investment policy outlined in the Proposition. We verify both conjectures once we have derived the pricing function.

Given that noise traders always trade a random amount \(z \sim U(-1, 1)\), we get the following four possibilities for total order flow \(X\): (i) if \(\theta_A = \theta_B = H\): \(X = \Gamma + z\), (ii) if \(\theta_A = H\) and \(\theta_B = L\): \(X = (2q_A - 1) \Gamma + z\), (iii) if \(\theta_A = L\) and \(\theta_B = H\): \(X = (1 - 2q_A) \Gamma + z\), and (iv) if \(\theta_A = \theta_B = L\): \(X = -\Gamma + z\). The market maker observes total order flow and sets the price equal to the expected future firm value \(E[V | X]\). It follows from the distributional assumption on \(z\) and \(q_A \in \left[\frac{1}{2}, 1\right]\), that the market maker can draw the following inferences:

1. If \(X \in (1 + (2q_A - 1) \Gamma, 1 + \Gamma)\): the market maker knows that \(\theta_A = \theta_B = H\) and that the manager will invest based on the price signal. It follows that the equilibrium price is equal to \(p_{HH} = x_A^H + x_B^H\).

2. If \(X \in (1 - (2q_A - 1) \Gamma, 1 + (2q_A - 1) \Gamma)\): the market maker knows that \(\theta_A = H\), that \(\theta_B \in \{L, H\}\) with equal probability, that the manager will invest in project \(A\) based on the price signal, and that the manager might invest in project \(B\) if \(\sigma_m = \theta_B = H\). It follows that the equilibrium price is equal to \(p^H = x_A^H + \frac{1}{2} \omega_B x_B^H\).

3. If \(X \in (1 - \Gamma, 1 - (2q_A - 1) \Gamma)\): the market maker knows that there are three equally likely scenarios \(\{\theta_A = H, \theta_B = H\}, \{\theta_A = H, \theta_B = L\}\), and \(\{\theta_A = L, \theta_B = H\}\). In the first case, the manager invests with probability \(\omega_A\) in project \(A\), in the second case, she invests with probability 1 because she either learns \(\theta_A = H\) from her private signal about \(A\) or her private signal about \(B\), in the third case, she invests with probability 1 in project \(B\) for the same
reason. It follows that the equilibrium price is equal to \( p^{0,H} = \frac{1}{3} \left( (1 + \omega_A)x_A^H + (1 + \omega_B)x_B^H \right) \).

4. If \( X \in (-1 + \Gamma, 1 - \Gamma) \): the market maker cannot learn anything about \( \theta_A \) and \( \theta_B \). He knows that the manager invests with probability \( \frac{1}{2} \omega_A \) and \( \frac{1}{2} \omega_B \) in the two projects. It follows that the equilibrium price is equal to \( p^{0,M} = \frac{1}{2} \left( \omega_A x_A^H + \omega_B x_B^H \right) \).

5. If \( X \in (-1 + (2q_A - 1)\Gamma, -1 + \Gamma) \): the market maker knows that there are three equally likely scenarios \( \{ \theta_A = H, \theta_B = L \}, \{ \theta_A = L, \theta_B = H \}, \) and \( \{ \theta_A = L, \theta_B = L \} \). In the first case, the manager invests in project A with probability \( \omega_A \), in the second case, the manager invests in project B with probability \( \omega_B \), in the third case the manager does not invest. It follows that the equilibrium price is equal to \( p^{0,L} = \frac{1}{3} \left( \omega_A x_A^H + \omega_B x_B^H \right) \).

6. If \( X \in (-1 - (2q_A - 1)\Gamma, -1 + (2q_A - 1)\Gamma) \): the market maker knows that \( \theta_A = L \), that \( \theta_B \in \{ L, H \} \) with equal probability. He knows that the manager does not invest in project A and that he invests in project B with probability \( \frac{1}{2} \omega_B \). It follows that the equilibrium price is equal to \( p^L = \frac{1}{2} \omega_B x_B^H \).

7. If \( X \in (-1 - \Gamma, -1 - (2q_A - 1)\Gamma) \): the market maker knows that \( \theta_A = \theta_B = L \) and that the manager does not invest in either project. It follows that the equilibrium price is equal to \( p^{LL} = 0 \).

Next, we have to verify the manager’s conjectured investment policy. If the manager observes \( \sigma_m = \theta_j = H \), the expected NPV of project \( j \) is equal to \( x_j^H > 0 \) and she optimally invests \( (K_j = 1) \). Next, we consider the cases in which the manager might invest based on a feedback signal:

1. If \( P = p^{HH} \): the manager knows \( \theta_A = \theta_B = H \) and optimally invests in both projects because \( \mathbb{E}[x_j^\theta | P = p^{HH}] = x_j^H > 0 \) for \( j \in \{ A, B \} \).

2. If \( P = p^H \): the manager knows that \( \theta_A = H \) and invests in project \( A \) because \( \mathbb{E}[x_A^\theta | P = p^H] = x_A^H > 0 \); he does not learn any new information about \( \theta_B \) from the price signal such that \( \mathbb{E}[x_B^\theta | P = p^H] = \frac{1}{2} (x_B^H + x_B^L) < 0 \) and does not invest in project \( B \).
3. If $P = p^0,H$: the manager knows that $\theta_A = H$ if $\sigma_m = \theta_B = L$ and that $\theta_B = H$ if $\sigma_m = \theta_A = L$.

As a result, she invests in project $A$ in the first case and project $B$ in the second case. In both cases $\mathbb{E}[x^\theta_j | P = p^0,H] = x^H > 0$ ($j = A$ in the first and $j = B$ in the second case).

Finally, we need to verify the conjectured trading policy. To this end, we first compute the expected stock price and firm value for the four different combinations of $\theta_A$ and $\theta_B$.

1. If $\theta_A = H$ and $\theta_B = H$, the equilibrium price can take on four different values, $p^{HH}$, $p^H$, $p^0,H$, and $p^0,M$ with the following probabilities:

   (a) $\mathbb{P}(P = p^{HH} | \theta_A = H, \theta_B = H) = (1 - q_A)\Gamma$

   (b) $\mathbb{P}(P = p^H | \theta_A = H, \theta_B = H) = (2q_A - 1)\Gamma$

   (c) $\mathbb{P}(P = p^0,H | \theta_A = H, \theta_B = H) = (1 - q_A)\Gamma$

   (d) $\mathbb{P}(P = p^0,M | \theta_A = H, \theta_B = H) = 1 - \Gamma$

Given the expressions for $P$ derived above, the expected stock price is equal to:

$$\mathbb{E}[P | \theta_A = H, \theta_B = H] = \frac{(3\omega_A + (2 - \omega_A)(2q_A + 1)\Gamma) x^H_A + (3(1 - \omega_A) + 4(1 + \omega_A)(1 - q_A)\Gamma) x^H_B}{6}.$$ 

The expected firm value is equal to:

$$\mathbb{E}[V | \theta_A = H, \theta_B = H] = (\omega_A + (1 - \omega_A)q_A\Gamma) x^H_A + (1 - \omega_A + \omega_A(1 - q_A)\Gamma) x^H_B.$$ 

2. If $\theta_A = H$ and $\theta_B = L$, the equilibrium price can take on four different values, $p^H$, $p^0,H$, $p^0,M$, and $p^0,L$ with the following probabilities:

   (a) $\mathbb{P}(P = p^H | \theta_A = H, \theta_B = L) = (2q_A - 1)\Gamma$

   (b) $\mathbb{P}(P = p^0,H | \theta_A = H, \theta_B = L) = (1 - q_A)\Gamma$

   (c) $\mathbb{P}(P = p^0,M | \theta_A = H, \theta_B = L) = 1 - \Gamma$

   (d) $\mathbb{P}(P = p^0,L | \theta_A = H, \theta_B = L) = (1 - q_A)\Gamma$

Given the expressions for $P$ derived above, the expected stock price is equal to:

$$\mathbb{E}[P | \theta_A = H, \theta_B = L] = \frac{(3\omega_A + 2(5q_A - 2)\Gamma + (1 - 4q_A)\omega_A\Gamma) x^H_A + (3(1 - \omega_A) + 2(1 - q_A)\omega_A\Gamma) x^H_B}{6}.$$ 

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The expected firm value is equal to:

\[ \mathbb{E}[V|\theta_A = H, \theta_B = L] = (\omega_A + (1 - \omega_A)q_A \Gamma) x_A^H. \]

3. If \( \theta_A = L \) and \( \theta_B = H \), the equilibrium price can take on four different values, \( p^{0,H} \), \( p^{0,M} \), \( p^{0,L} \), and \( p^L \) with the following probabilities:

(a) \( \mathbb{P}(P = p^{0,H}|\theta_A = L, \theta_B = H) = (1 - q_A)\Gamma \)

(b) \( \mathbb{P}(P = p^{0,M}|\theta_A = L, \theta_B = H) = 1 - \Gamma \)

(c) \( \mathbb{P}(P = p^{0,L}|\theta_A = L, \theta_B = H) = (1 - q_A)\Gamma \)

(d) \( \mathbb{P}(P = p^L|\theta_A = L, \theta_B = H) = (2q_A - 1)\Gamma \)

Given the expressions for \( P \) derived above, the expected stock price is equal to:

\[ \mathbb{E}[P|\theta_A = L, \theta_B = H] = \frac{(3\omega_A + (2 + \omega_A)/2 - 2(1 + 2\omega_A)q_A \Gamma) x_A^H + (3(1 - \omega_A) + 2(1 - q_A)\omega_A \Gamma) x_B^H}{6}. \]

The expected firm value is equal to:

\[ \mathbb{E}[V|\theta_A = L, \theta_B = H] = (1 - \omega_A + \omega_A(1 - q_A)\Gamma) x_B^H. \]

4. If \( \theta_A = L \) and \( \theta_B = L \), the equilibrium price can take on four different values, \( p^{0,M} \), \( p^{0,L} \), \( p^L \), and \( p^{LL} \) with the following probabilities:

(a) \( \mathbb{P}(P = p^{0,M}|\theta_A = L, \theta_B = H) = 1 - \Gamma \)

(b) \( \mathbb{P}(P = p^{0,L}|\theta_A = L, \theta_B = H) = (1 - q_A)\Gamma \)

(c) \( \mathbb{P}(P = p^L|\theta_A = L, \theta_B = H) = (2q_A - 1)\Gamma \)

(d) \( \mathbb{P}(P = p^{LL}|\theta_A = L, \theta_B = H) = (1 - q_A)\Gamma \)

Given the expressions for \( P \) derived above, the expected stock price is equal to:

\[ \mathbb{E}[P|\theta_A = L, \theta_B = L] = \frac{(3(1 - q_A)\Gamma) \omega_A x_A^H + (3 - 4(1 - q_A)\Gamma) (1 - \omega_A)x_B^H}{6}. \]

The expected firm value is equal to:

\[ \mathbb{E}[V|\theta_A = L, \theta_B = L] = 0. \]

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Next, we compute the ex ante trading profits for an individual trader.

\[
\mathbb{E}[\Pi_i|\sigma_i = \theta_A = H] = \left( \frac{3\omega_A + 1}{3} x_A^H + \frac{2}{3} x_B^H \right) \frac{(1 - q_A)\Gamma}{2} + \omega_A \frac{1 - \Gamma}{2} x_A^H
\]

\[
\mathbb{E}[\Pi_i|\sigma_i = \theta_A = L] = \left( \frac{3\omega_A + 1}{3} x_A^H + \frac{2}{3} x_B^H \right) \frac{(1 - q_A)\Gamma}{2} + \omega_A \frac{1 - \Gamma}{2} x_A^H
\]

\[
\mathbb{E}[\Pi_i|\sigma_i = \theta_B = H] = (1 - 2(1 - q_A)\Gamma) \frac{1}{2} x_B^H + \left( \frac{4 - 3\omega_A}{3} x_B^H - \frac{2}{3} x_A^H \right) \frac{(1 - q_A)\Gamma}{2}
\]

\[
\mathbb{E}[\Pi_i|\sigma_i = \theta_B = L] = (1 - 2(1 - q_A)\Gamma) \frac{1}{2} x_B^H + \left( \frac{4 - 3\omega_A}{3} x_B^H - \frac{2}{3} x_A^H \right) \frac{(1 - q_A)\Gamma}{2}
\]

These expected trading profits are positive for \(A\)-informed and \(B\)-informed traders under the following conditions:

1. If \(x_B^H \in \left( 0, \frac{1}{2} x_A^H \right)\), we require \(\omega_A \in \left( 0, \frac{3x_B^H - 2(1 - q_A)(x_A^H + x_B^H)}{3(1 - (1 - q_A)\Gamma)x_A^H} \right)\) and either \(\Gamma \in \left( 0, \frac{3x_B^H}{x_A^H + x_B^H} \right)\), \(q_A \in \left[ \frac{1}{2}, 1 \right)\) or \(\Gamma \in \left[ \frac{3x_B^H}{x_A^H + x_B^H}, 1 \right], q_A \in \left[ \frac{1}{2}, 1 \right]\).

2. If \(x_B^H \in \left( \frac{1}{2} x_A^H, x_A^H \right)\), we require \(\Gamma \in (0, 1), q_A \in \left[ \frac{1}{2}, 1 \right]\), and \(\omega_A \in \left[ \frac{(1 - q_A)(x_B^H - x_A^H)\Gamma}{2(1 - (1 - q_A)\Gamma)x_A^H}, \frac{3x_B^H - 2(1 - q_A)(x_A^H + x_B^H)}{3(1 - (1 - q_A)\Gamma)x_B^H} \right]\).

When we derive the optimal information choices \(\omega_A\) and \(q_A\), we will show that these conditions are always fulfilled.

**A.2.2 Proof of Proposition 2**

The first result for \(\omega_A^*\) follows directly from equation (7).

For the second result, we have to characterize the value of \(q_A^*\) in the intermediate range. To this end, note that in equation (9) the expression \(\frac{\Gamma - x_A^H - \omega_A(1 + (1 - \Gamma)x_A^H)}{\Gamma(1 + \omega_A(1 - x_A^H))} \in (0, 1)\) in the intermediate range and that it is in increasing in \(\omega_A\).

**A.2.3 Proof of Proposition 3**

First, we compute the trader’s ex ante expected profit based on receiving \(\sigma_i = \theta_A\) and \(\sigma_i = \theta_B\) based on the expressions derived in Appendix A.2.1.

\[
\mathbb{E}[\Pi_i|\sigma_i = \theta_A] = \frac{1}{6} \left( 2\Gamma (q_A - 1) x_B^H + x_A^H (-q_A(3\Gamma \omega_A + \Gamma) + \Gamma + 3\omega_A) \right)
\]

and

\[
\mathbb{E}[\Pi_i|\sigma_i = \theta_B] = \frac{1}{6} \left( x_B^H (\Gamma(2 - 3\omega_A)q_A + (\Gamma - 1)\omega_A - 2\Gamma + 3) + 2\Gamma (q_A - 1) x_A^H \right).
\]
Setting these two expressions equal to each other leads to:

\[ q_A = \frac{x_A^H(\Gamma + \omega_A) + x_B^H(-\Gamma \omega_A + \omega_A - 1)}{\Gamma(\omega_A + 1)x_A^H - \Gamma \omega x_B^H}. \]

As we show in the text, the manager’s information acquisition problem has an interior solution if \( q_A = \frac{x_A^H+(-1)x_B^H}{\Gamma(x_A^H+x_B^H)}. \) Finally, we set these two expressions equal to each other and solve for \( \omega_A. \) It follows that the resulting choices are between 0 and 1 if \( \Gamma > 1 - \kappa_x. \) If \( \Gamma < 1 - \kappa_x, \) we get a corner solution and \( \omega_A^* = q_A^* = 1. \) In the knife-edge case, \( \Gamma = 1 - \kappa_x, \) traders choose \( q_A = 1 \) and the manager is indifferent between any \( \omega_A \in [\frac{1}{2}, 1]. \) We assume that, when indifferent, all agents pursue pure-strategy equilibria such that \( \omega_A^* = 1 \) in this case.

### A.2.4 Proof of Proposition 4

First, note that our measure of market efficiency is given by, \( ME = \frac{\mathbb{E}[\text{Var}(V|P)]}{\text{Var}(P)}. \) As we explain in the text, we can show that market efficiency is equivalent to \( \frac{\text{Var}(P)}{\text{Var}(V)} - 1. \) Next, we compute the unconditional variance of the firm’s stock price and long-run value based on our results in Proposition 1:

\[
\text{Var}(P) = -\frac{1}{12} \Gamma \left((q_A-1)(x_B^H)^2(3\Gamma \omega_A^2 q_A + (2 - 3\Gamma)\omega_A^2 + 2) \right) \\
- \frac{1}{12} \Gamma \left(2(q_A-1)x_A^H x_B^H(3\Gamma(\omega_A - 1)\omega_A q_A - 2(\omega_A - 1)\omega_A + 2) \right) \\
- \frac{1}{12} \Gamma \left((x_A^H)^2(q_A(3\Gamma(\omega_A - 1)^2q_A - 4(\omega_A - 2)\omega_A - 8) + (\omega_A - 2)\omega_A + 2) \right)
\]

and

\[
\text{Var}(V) = \frac{1}{2} \left(\Gamma x_A^H x_B^H((1 - 2\omega_A)q_A + \omega) - (x_B^H)^2(\Gamma \omega_A q_A - \Gamma \omega_A + \omega_A - 1) + (x_A^H)^2(q_A(\Gamma - \Gamma \omega_A) + \omega_A) \right) \\
- \frac{1}{4} \left(\omega_A x_B^H(\Gamma (-q_A) + \Gamma - 1) + x_A^H(q_A(\Gamma - \Gamma \omega_A) + \omega_A) + x_B^H \right)^2.
\]

Then we combine these two expressions and plug in the equilibrium values for \( q_A \) and \( \omega_A \) from Proposition 3 to obtain the expression for \( ME \) in Proposition 4. It is straightforward to show that \( ME^* \) is strictly increasing in \( \Gamma. \) For the comparative statics with respect to \( \kappa_x, \) we have to consider three different cases.

1. \( \Gamma \in (0, 0.180297], \) \( \frac{\partial ME^*}{\partial \kappa_x} \geq 0 \) for all \( \kappa_x \in (1 - \Gamma, 1]; \)
2. If \( \Gamma \in (0.180297, 1) \): \( \frac{\partial ME^*}{\partial \kappa} < 0 \) if \( \kappa_x \in (1 - \Gamma, \hat{\kappa}) \) and \( \frac{\partial ME^*}{\partial \kappa} \geq 0 \) if \( \kappa_x \in (\hat{\kappa}, 1] \); \( \hat{\kappa} \) is the solution to the following equation:

\[
0 = \kappa_x^{12} + \kappa_x^{11} (5 - \Gamma) + \kappa_x^{10} (-2\Gamma^2 + 4\Gamma + 4) + \kappa_x^9 (7\Gamma^3 - 46\Gamma^2 + 86\Gamma - 43) \\
+ \kappa_x^8 (8\Gamma^4 - 45\Gamma^3 + 55\Gamma^2 + 25\Gamma - 49) + \kappa_x^7 (4\Gamma^5 - 36\Gamma^4 + 140\Gamma^3 - 345\Gamma^2 + 488\Gamma - 286) \\
+ \kappa_x^6 (6\Gamma^3 - 48\Gamma^2 + 132\Gamma - 120) + \kappa_x^5 (-3\Gamma^5 + 32\Gamma^4 - 133\Gamma^3 + 317\Gamma^2 - 394\Gamma + 186) \\
+ \kappa_x^4 (-5\Gamma^5 + 36\Gamma^4 - 98\Gamma^3 + 166\Gamma^2 - 198\Gamma + 111) + \kappa_x^3 (-3\Gamma^5 + 24\Gamma^4 - 72\Gamma^3 + 109\Gamma^2 - 95\Gamma + 41) \\
+ \kappa_x^2 (2\Gamma^5 - 14\Gamma^4 + 38\Gamma^3 - 54\Gamma^2 + 40\Gamma - 12) + \kappa_x (-2\Gamma^3 + 5\Gamma^2 - 4\Gamma + 1) - \Gamma^3 + 3\Gamma^2 - 3\Gamma + 1
\]

It can be shown that \( \hat{\kappa} \in (1 - \Gamma, 1) \).

3. \( \Gamma = 1 \): \( \frac{\partial ME^*}{\partial \kappa} < 0 \) if \( \kappa_x \in (0, 0.640316) \) and \( \frac{\partial ME^*}{\partial \kappa} \geq 0 \) if \( \kappa_x \in [0.640316, 1] \).

Second, our measure of real efficiency is the expected long-term firm value. To compute this measure we take the unconditional expectation of \( V \) based on the results in Proposition 1:

\[
\mathbb{E}[V] = \frac{1}{2} (\omega_A x_B^H (\Gamma (-q_A) + \Gamma - 1) + x_A^H (q_A (\Gamma - \Gamma \omega_A) + \omega_A) + x_B^H).
\]

Then we plug in the equilibrium values for \( q_A \) and \( \omega_A \) from Proposition 3 to get the expression for \( RE \) in Proposition 4. It immediately follows that \( \frac{\partial RE^*}{\partial \kappa} \geq 0 \) and \( \frac{\partial RE^*}{\partial \Gamma} \geq 0 \).

A.2.5 Proof of Corollary 1

First, we consider real efficiency for a given level of \( \Gamma \in (0, 1] \). We know from the results in Proposition 4 that \( RE \) is minimized at \( \frac{1}{2} x_A^H \) for any \( \kappa_x \leq 1 - \Gamma \). Then, we also know that for \( \kappa_x > 1 - \Gamma \), \( RE \) is strictly increasing in \( \kappa_x \) which implies that it reaches its highest value (for a fixed \( \Gamma \)) at \( \kappa_x = 1 \).

Next, we consider market efficiency for a given level of \( \Gamma \in (0, 1] \). First note that for any \( \kappa_x \leq 1 - \Gamma \), \( ME = \Gamma - 1 \). Then, consider the limit \( \lim_{\kappa_x \uparrow 1-\Gamma} ME = \frac{(4-\Gamma)(1-\Gamma)}{4} = \Gamma - 1 + \frac{\Gamma(1-\Gamma)}{4} \geq \Gamma - 1 \). Thus, we know that \( ME \) is maximized for some \( \kappa_x \in (1 - \Gamma, 1] \). Moreover, we can use the comparative statics with respect to \( \kappa_x \) from Proposition 4 which imply that \( ME^* \) is either \( U \)-shaped or weakly increasing (in \( \kappa_x \)) in this region.
Thus we just need to compare two points in the $\kappa_x \in (1 - \Gamma, 1]$ region: $\kappa_x \downarrow 1 - \Gamma$ and $\kappa_x = 1$. Evaluating $ME^*$ at these two values implies that it is maximized at $\kappa_x \downarrow 1 - \Gamma$ for $\Gamma \leq \tilde{\Gamma} = 0.403219$ and maximized at $\kappa_x = 1$ if $\Gamma \geq \tilde{\Gamma} = 0.403219$.

For the smallest value of market efficiency, we can also focus on two cases. First, any $\kappa_x \in (0, 1 - \Gamma]$ that leads to $ME^* = \Gamma - 1$ and second, $\kappa_x = \tilde{\kappa}$ if $\Gamma \in (0.180297, 1)$. It follows that:

1. $\Gamma \in (0, 1)$: $ME^*$ is minimized at $\kappa_x \leq 1 - \Gamma$.

2. $\Gamma = 1$: ME is minimized at $\kappa_x = 0.640316$.

### A.2.6 Proof of Proposition 5

First, we show that the trading and investment equilibrium in Proposition 1 is unaffected by the manager’s dismissiveness. To this end, we conjecture that traders follow the same trading policy as before and buy (sell) on good (bad) news about $\theta$. It follows that actual total order flow is to that in the main model. From the manager’s perspective $\Gamma$ is replaced by $\lambda \Gamma$. As before, there are seven distinct regions for $X$ such that the market maker sets seven distinct prices, $P(X)$. In particular, the manager learns the same information from $P$, she simply underestimates the probability with which the price is informative, i.e. $P \neq p^{0,M}$.

Second, we derive the traders’ and the manager’s best-response functions. Since the manager assumes that all agents use $\lambda \Gamma$ instead $\Gamma$, the optimal value for $\omega_A$ is identical to that in Proposition 3 with $\Gamma = \lambda$. Traders are fully rational and aware of the manager’s dismissiveness. Plugging in the manager’s information choice into the expression for expected profits derived above yields to the expressions for $\tilde{q}_A$ given in Proposition 5.

### A.2.7 Proof of Proposition 6

First, we use the definition of real efficiency as the expected firm value $\overline{\mathbb{E}}[V]$. Then, we plug in the optimal information choices $\tilde{\omega}_A$ and $\tilde{q}_A$ derived in Proposition 5.

It follows that real efficiency is equal to $\frac{1}{2} \chi_A^H$ if $\lambda \leq 1 - \kappa_x$. If $\lambda > 1 - \kappa_x$, real efficiency is equal to:

$$
\tilde{RE} = \frac{(\kappa_x((\kappa_x - 2)\kappa_x^2 + (\kappa_x - 1)\kappa_x \lambda - 3 \lambda^2 + 11 \lambda - 10) + \lambda - 1) \chi_A^H}{2(\kappa_x + 1)(2 - \lambda)((\kappa_x - 2)\kappa_x + \lambda - 1)}.
$$
It is straightforward to show that this expression exceeds $\frac{1}{2}x_{A}^{H}$, which implies that firm is always better off hiring a manager with $\lambda > 1 - \kappa_{x}$.

In a last step, we can take the first and second derivative of $\tilde{RE}$ to solve for the optimal $\lambda$ in the interval $(1 - \kappa_{x}, 1)$. 