We thank workshop participants at LSE and faculty at UCLA and Wharton and for helpful discussions. The usual caveat applies.
Interested Intermediaries

Abstract: We examine the implications of interested intermediaries on corporate actions, stock prices, and investors' portfolio decisions. An interested intermediary is an asset manager who has private preferences over corporate actions, which could relate to corporate governance policies, social or environmental performance, payout policy, etc. We find that free riding on beneficial influence activities can drive out intermediaries, but their presence is restored by investing frictions, information asymmetry, or the potential for investors to make behavioral mistakes. Intermediaries seek to economize on costly influence efforts, which is facilitated by having interests aligned with managers. Endogenous delegation with interested intermediaries can cause economic features of the environment to have opposite-signed effects on stock price and expected firm cash flows. Our findings have implications for research and policy-making related to asset management, investor activism, and corporate social responsibility.
1 Introduction

Vast amounts of wealth are managed by investment intermediaries. Assets under management topped $85 trillion as of the end of 2017 (Baghai et al., 2018). Because of their holdings, intermediaries can influence firms through voting, voice, and portfolio choices (e.g., Dimson et al., 2015; McCahery et al., 2016).\(^1\) This influence can be wielded in a disinterested manner to maximize portfolio risk-adjusted returns, but can also be used to advance the interests of the intermediary or their investors. Prior evidence suggests that investors, including intermediaries, have such preferences (e.g., Gantchev et al., 2018; Li and Raghunandan, 2019).\(^2\)

In this paper, we study the implications of interested intermediaries, who make portfolio decisions on behalf of their client investors but have preferences over the actions that firms take. Our analysis centers on a series of models. Our main model starts with a continuum of atomistic investors who purchase shares in a firm and then can exert influence efforts that affect the action the firm’s manager takes. Through their influence, investors can, indirectly, increase the firm’s cash flows. Despite this potential, these direct investors optimally exert no influence efforts. Because their holdings are small, the benefits to them are negligible relative to the private cost of cash-flow improving effort.

We next introduce an intermediary, i.e., an asset manager. The asset manager makes portfolio decisions on behalf of a measurable fraction of investors, but also has private preferences for the action the manager takes. This preference can be for actions associated with higher expected cash flows (e.g., beneficial governance attributes) or lower expected cash

\(^1\)Asset managers, even passive ones, have a legal duty of care to their investor clients that requires them to vote their clients’ proxies (https://www.sec.gov/rules/final/ia-2106.htm#P55_8940). Large asset managers “maintain dedicated investment stewardship teams, which independently develop their own guidelines for engagement and voting.” (Mallow, 2019, p. 10)

\(^2\)Sustainable investment funds topped $30 trillion as of the beginning of 2018. (GSIA, 2018) Furthermore, the U.K. Financial Reporting Council recently amended its stewardship code to require investment managers to consider environmental, social, and governance (ESG) factors when making investments. While not mandatory, the Financial Conduct Authority requires that asset managers either comply with the stewardship code or explain why they do not. (Trentmann, 2019)
flows (e.g., costly emissions reductions). The asset manager exerts non-negligible influence efforts, both because of its private interest in the firm’s action and its desire to improve the portfolio outcome for its client investors. In equilibrium, the firm’s stock price is increasing in the fraction of investors who delegate to the intermediary, because this increases the intermediary’s incentives to exert cash-flow improving influence effort. The effect of making influence activities costlier, however, depends on the intermediary’s interests. If the intermediary prefers more positive managerial actions or is relatively aligned with the manager’s preference, an increase in cost lowers the intermediary’s efforts. Conversely, if the intermediary prefers more negative actions (capturing, e.g., costly corporate social responsibility or CSR investments), then an increase in costs weakens the intermediary’s influence and allows the firm to pursue greater cash flows, increasing stock price.

Interestingly, the free-rider problem between atomistic investors re-emerges in the setting with an intermediary, but in a subtler form. When we endogenize the fraction of investors who delegate, we find that it is optimal for all investors not to delegate. Because the intermediary’s influence efforts affect the firm’s expected cash flows, direct investors, those that do not delegate, also benefit from the influence activities taken by the intermediary. However, they do not bear the costs of the influence efforts, which are borne only by the investors who delegate their portfolio decisions to the intermediary. Thus, investing through the intermediary is net costly, and no investors choose to do so in equilibrium. Free riding on the intermediary and the delegating investors thus pushes the equilibrium back to that found without the intermediary, in which all investors invest directly in the firm and exert no costly influence efforts.

To arrive at an interior optimal degree of delegation, we first introduce a fixed cost of direct investment, as in Admati et al. (1994), which represents transaction costs direct investors face that can be avoided by portfolio delegation. We find that the fraction of investors who delegate is decreasing in the divergence between the intermediary’s and manager’s preferred actions. Intermediaries benefit from preference alignment because it weakens their incentives
for costly influence efforts. This lowers the costs borne by delegating investors. For the same reason, we also find that an intermediary whose preferences are aligned benefits from having strong private preferences over the manager’s action, as this further decreases the relative desire to exert influence efforts to increase cash flows. Jointly, these results provide a novel justification for the evidence that intermediaries are concerned about corporate governance (McCahery et al., 2016) but frequently vote with management (e.g., Heath et al., 2019).3

With endogenous delegation, furthermore, an increase in the costs of direct investing tends to increase the firm’s stock price. This is a potentially counterintuitive effect, but evolves naturally from the forces in our model. Greater costs increase the fraction of delegating investors. This increases the intermediary’s incentive to exert cash-flow increasing influence efforts, which in turn increase cash flows. A decrease in retail investor costs, therefore, can have a negative effect on stock price because it facilitates free-riding on investor influence efforts.

In extensions, we examine the implications of additional forces that may be at play. First, we allow for investors to have preferences over firms’ actions (e.g., Bénabou and Tirole, 2010; Fama and French, 2007; Friedman and Heinle, 2016). Despite their private benefit from affecting managerial actions, they continue to optimally exert no influence efforts. Delegation here, however, causes the intermediary to internalize investors’ preferences. A greater share of delegating investors causes investors’ preferences to be better reflected in the manager’s action. With endogenous delegation, however, changes in investors’ preferences have non-monotonic effects on the fraction of investors who choose to delegate portfolio decisions to the intermediary. As before, this effect largely operates through the effects of preferences and interests on the magnitude of costly influence efforts undertaken by the intermediary, with fewer investors delegating when the intermediary is expected to exert more costly effort.

Our second extension examines the effects of an insider who owns a fixed fraction of

3BlackRock, a large asset manager, votes at “more than 17,000 shareholder meetings globally each year, on over 160,000 ballot items,” and their “starting position is to support management unless severe governance or performance concerns are identified.” (Novick et al., 2018, p. 9)
the firm, has private interests in the manager’s action, and can also exert costly influence efforts. The presence of the insider changes the influence activities of the intermediary. In the Nash equilibrium to the influence subgame, they anticipate each others’ influence efforts when choosing their own. Greater holdings by the insider imply greater influence efforts from her, allowing the intermediary to reduce its influence efforts and economize on those costs. Because of this, an increase in the fraction of the firm held by the insider, while reducing the shares held by direct investors, can either increase or decrease the equilibrium holdings of delegating investors.

The third extension provides a microeconomic foundation for the cost of direct investing, based on the intermediary having an information advantage. As in Marinovic and Varas (2019), we assume the intermediary’s costs of influence are ex ante random, but privately observed by the intermediary prior to trading. The randomness in the intermediary’s influence costs imply randomness in the optimal efforts expended and thus randomness in the firm’s cash flows. This imposes additional risk on direct investors, which can be avoided via delegation. We find that the expected stock price is decreasing in the variance of the intermediary’s costs, due to reduced demand from direct investors. This, however, leaves more of the firm held by the intermediary, which leads to more positive influence efforts and cash flows in expectation.

Our fourth extension also introduces an information asymmetry between delegating and direct investors, but does so by assuming that direct investors respond to noisy signals as if they are informative, due to, e.g., sentiment, mood, or overconfidence (e.g., Bushee and Friedman, 2016). In equilibrium, direct investors underestimate the riskiness of cash flows, which increases their demand and reduces the shares held by the intermediary. This lowers influence efforts and expected cash flows, but the positive effect on demand yields a higher expected stock price.

The study contributes broadly to our understanding of investors’ influence efforts, delegation choices, and the effects of preferences over managerial actions. Our framework and
results can help inform research and policy-making related to asset management, investor activism, and corporate social responsibility.

2 The Model

There is a firm, $f$, with a manager who takes an action $a_f$. The firm’s cash flows per share are normally distributed such that

$$x_f = \beta a_f + \varepsilon_f.$$  

The mean is $E[x_f] = \beta a_f$, where $a_f$ is the action the manager takes and $\beta > 0$ is the impact of firm $f$’s manager’s action on firm $f$’s expected cash flow. The noise term is normally distributed with mean zero and variance $\sigma_f^2$. We assume that the firm is traded on a competitive market and that the supply of shares equals 1. There is a continuum of risk averse investors of mass 1, each with CARA utility function,

$$u = -\exp(-\tau W),$$

such that the aggregate risk aversion in the economy is equal to $\tau$.

2.1 Timeline

The timeline consists of three periods. In the first period, investors trade. In the second period, investors can influence the manager via efforts, $m_j$, for investor $j \in [0,1]$. The manager then takes the action $a_f$. In period 3, the firm’s cash flows are realized and all
parties consume.

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<td>trading, $p$</td>
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**Figure 1:** Timeline

### 2.2 Manager’s actions

The manager chooses her action, $a_f$, to maximize the following utility function

$$u_{fm} = -(a_f - m_f - \mu_f)^2,$$  \hspace{1cm} (1)

where $a_f$ is the action taken by manager $f$, $m_f = \int m_j dj$ is the aggregate influence/monitoring undertaken in the economy, and $\mu_f$ is the action that the manager would take in the absence of activist influence. This leads to an action choice of

$$a_f = m_f + \mu_f.$$  \hspace{1cm} (2)

Each investor has utility

$$CE^m_j = q_j (\beta a_f - p_f) - \frac{1}{2} \tau q_f^2 \sigma_f^2 - \frac{c}{2} m_j^2.$$  \hspace{1cm} (3)

### 2.3 No intermediaries

Without intermediaries, $m_f = \int_{j=0}^{1} m_j dj$. The effect of a single investor on the manager’s action is negligible, as $\frac{da_f}{dm_j} = dj \approx 0$. However, the cost from an atomistic investor’s perspective is $cm_j \geq 0$. Atomistic investors therefore optimally exert no influence effort, as their efforts have positive cost but infinitesimal benefit because their marginal effects on cash flows and efforts are also atomistic. That is, small investors hold small portfolios and
have small effects, even though they hold the entire firm and could provide a measurable level of influence in aggregate. This captures the fundamental free-rider problem of investor activism. Costly influence/monitoring efforts yield a social benefit but private costs, such that it is optimal for each small investor to exert no influence/monitoring efforts, even if they disagree with the actions taken by the manager or can improve cash flows by exerting efforts. We summarize this result in Lemma 1 below.

**Lemma 1** Atomistic investors find it optimal to exert no influence efforts.

With a continuum of small investors, investors’ preferences have no effect on the manager’s action or the firm’s cash flows. Prior studies have overcome this null result by assuming either: a) investors coordinate (Oehmke and Opp, 2019); b) there is a representative, i.e., single shareholder (Chowdhry et al., 2018; Morgan and Tumlinson, 2019); or c) investors get “warm glow” utility from holding shares and managers care about stock price (Friedman and Heinle, 2016; Pástor et al., 2019; Zivin and Small, 2005). Our result in Lemma 1 highlights that these assumptions may be necessary for investor preferences to affect corporate actions. Even if investors care about corporate actions, they are better off if someone else bears the cost of inducing those actions, and this free riding can in equilibrium sever the tie between preferences and induced actions.

### 2.4 An interested intermediary

We next introduce an intermediary who can take costly influence efforts on behalf of its portfolio investors, but also may be interested separately in the actions the manager takes, \(a_f\). The fraction of investors who delegate to the intermediary is \(\lambda \in [0, 1]\).\(^4\) Without loss of generality, we order the (homogeneous) investors such that this mass of delegators is the first \(\lambda\), i.e., investors \(j \in [0, \lambda]\) delegate to the intermediary while investors \(j \in (\lambda, 1]\) select

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\(^4\)The exogenously delegating investors could, for instance, be investors who gain “warm glow” utility from affecting corporate actions and understand that such effects are only feasible via intermediated coordination. Alternatively, they could represent investors with social goals (e.g., pension funds seeking to improve working conditions, religious or university endowments, or sovereign wealth funds).
their portfolios directly. At this stage, we take $\lambda$ as given, deferring the derivation of an equilibrium $\lambda$ to a later section.

Conditional on $\lambda$ investors delegating to the intermediary, $i$, maximizes the following utility function:

$$
\begin{align*}
    u_i &= \int_{j=0}^{\lambda} CE_j d\lambda - \frac{\gamma}{2} (a_f - A_{i,f})^2 - \frac{cm_i^2}{2} \\
    &= \lambda CE_j - \frac{\gamma}{2} (a_f - A_{i,f})^2 - \frac{cm_i^2}{2},
\end{align*}
$$

(4)

where $CE_j = q_i (\beta a_f - p_f) - \frac{1}{2} \tau q_i^2 \sigma_f^2$ is the certainty equivalent of an investor who delegates their investment choice to the intermediary and (optimally) chooses a personal influence effort of zero, $m_j = 0$. $A_{i,f}$ is the action that the intermediary/activist would prefer the firm to take. This captures intermediary preferences for, e.g., corporate social responsibility (CSR), governance, or other aspects of the firm not directly reflected in cash flows, $x_f$.

The parameters, $\gamma \geq 0$ and $c > 0$ capture, respectively, the costs to the intermediary of the manager’s action deviating from the intermediary’s preference and the direct cost of influence activities. A higher $\gamma$ implies the intermediary cares more about the manager’s action, all else equal (i.e., relative to the firm’s cash flows). Taken together, we assume that the intermediary acts in the interest of all delegating investors but faces a private cost of effort and has preferences over the action that the manager takes.

We next derive optimal influence, share demand, and stock price via backward induction. At $t = 2$, the interested intermediary, who we also refer to as an activist, has acquired aggregate holdings of $q_t$ in the firm on behalf of investors $j \in [0, \lambda]$ and the price paid to acquire the shares is sunk. Therefore, the activist will choose the following influence:

$$
    m_t^* \in \arg \max_{m_t} \lambda \left( q_t (\beta a_f - p_f) - \frac{1}{2} \tau q_t^2 \sigma_f^2 \right) - \frac{\gamma}{2} (a_f - A_{i,f})^2 - \frac{cm_t^2}{2}.
$$
Solving the first-order condition yields influence as a function of shares held,

\[ m_i^*(q_i) = \frac{\gamma (A_{i,f} - \mu_f)}{c + \gamma} + \frac{\beta \lambda q_i}{c + \gamma}. \]  

(5)

The intermediary’s optimal influence has two components. The first component is independent of the intermediary’s share holdings, \( q_i \). This component reflects the intermediary’s preference over the manager’s actions from the intermediary’s direct interests. When the intermediary has a preference for higher actions relative to the manager (i.e., \( A_{i,f} > \mu_f \)), the intermediary will provide more influence effort. Note that the intermediary’s influence can be negative, i.e., \( m_i^* \in \mathbb{R} \). This happens when the intermediary’s direct preference for the manager’s action is sufficiently below the manager’s preference, i.e., \( A_{i,f} << \mu_f \). This could, for instance, reflect an intermediary who objects to certain business practices that, while profitable, may be socially undesirable (e.g., related to worker safety or environmental policies).

The second component in equation (5) depends on the fraction of investors who delegate, \( \lambda \), on the intermediary’s holdings, \( q_i \), and on the efficacy of influence efforts, \( \beta \). This component reflects the cash flow implications of influence. A higher fraction of investors who delegate, \( \lambda \), implies that the intermediary acts on behalf of more shareholders and, therefore, has stronger incentives to increase cash flows. Similarly, higher \( q_i \) increases the incentives to provide influence because the intermediary has a larger benefit from the increased cash flows. Greater efficacy means that every unit of influence will have a more positive effect on cash flows, further increasing the benefit of influence efforts to the delegating investors.

The coefficient on the cost of influence, \( c \), and the coefficient on the preference divergence, \( \gamma \), deflate the impacts of both additive components in equation (5). However, while an increase in \( c \) strictly leads to lower influence effort, an increase in \( \gamma \) makes a divergence in preferences more important. As the importance of the intermediary’s preference gets large (i.e., as \( \gamma \rightarrow \infty \)), the intermediary tends to choose an influence that induces the manager
to choose an effort at the intermediary’s bliss point (i.e., such that $a_f \rightarrow A_{i,f}$). As the intermediary loses direct interest in the manager’s action, i.e., as $\gamma \rightarrow 0$, the intermediary shifts to choosing influence efforts only to maximize its investors’ cash flows, net of influence costs.

Substituting the intermediary’s influence from (5) into (4) yields their expected utility at $t = 1$, solving the FOC for maximizing the expected utility over the demand of shares in the firm then yields

$$q_i = \frac{\beta e^{\mu_f + \gamma A_{i,f}} - p_f}{\tau \sigma_f^2 - \frac{\beta^2 \lambda}{c + \gamma}},$$

(6)

which is a maximum for $\tau \sigma_f^2 - \frac{\beta^2 \lambda}{c + \gamma} > 0$, which we assume. If $\tau \sigma_f^2 - \frac{\beta^2 \lambda}{c + \gamma} < 0$, then the intermediary’s expected utility is everywhere increasing in $q_i$, such that the intermediary will wholly own the firm.

Similar to models without influence, the intermediary will demand fewer shares when the price of the firm, $p_f$, is higher, when cash flows are riskier (i.e., when $\sigma_f^2$ is higher), or when the investors that delegate are more risk averse (higher $\tau$). Different from models without influence, the numerator in the demand function is not given by the difference between expected cash flows and price. The reason is that with influence, the intermediary’s demand itself has an effect on the expected cash flows. Solving for demand therefore requires removing the demand component of expected cash flows. However, the impact of demand on cash flows then shows up in the denominator. Holding price fixed (because we have not yet solved for the equilibrium price), higher demand provides increased incentives to take an influence action that increases the manager’s effort and, thus, increases cash flows. Therefore, a higher impact of the influence on cash flows (through the productivity parameter, $\beta$) or an increased importance of cash flows to the intermediary (through the fraction of delegating investors, $\lambda$) increase demand. This is counteracted by the cost of effort, $c$, or the importance of the intermediary’s preference, $\gamma$, because these two parameters reduce the effect of demand on influence taking.
Anticipating the intermediary’s demand and influence, a direct investor’s demand is given by

\[ q^*_j = \frac{E[x] - p_f}{\tau \sigma^2_f}, \quad (7) \]

where \( E[x] = \beta \left( \frac{1}{c+\gamma} \left( \gamma (A_{i,f} - \mu_f) + \beta \frac{\mu_f + \gamma A_{i,f}}{c+\gamma} - p_f \right) + \mu_f \right) \). That is, the demand of a direct trader in a setting where an intermediary takes an influence action takes the same form as in a setting without influence. Substituting \( E[x] \) and simplifying terms yields

\[ q^*_j = \frac{\beta (\mu_f + \gamma A_{i,f})}{c+\gamma} - \frac{p_f}{\tau \sigma^2_f} - \frac{\beta \lambda}{c+\gamma} \text{ or } q^*_j = q^*_i. \quad (8) \]

Surprisingly, a direct investor has exactly the same demand as a delegating investor has via the intermediary. While, for example, a higher cost of effort does not directly change a direct investor’s utility, it does change the expected cash flow of the firm. As a result, even a direct investor who does not monitor internalizes the cost of monitoring.

The price is therefore defined by market clearing as \( 1 = \lambda q_i + (1 - \lambda) q_j \) or \( 1 = q^*_j \). This condition implies

\[ p_f = \frac{\beta^2 \lambda + \beta \left( c \mu_f + \gamma A_{i,f} \right)}{c+\gamma} - \tau \sigma^2_f. \quad (9) \]

The optimal monitoring action from the intermediary is thus

\[ m^*_i = \frac{\beta \lambda + \gamma (A_{i,f} - \mu_f)}{c+\gamma}. \quad (10) \]

In equilibrium, the intermediary’s monitoring action is a weighted average of the impact on cash flows (through the productivity of effort, \( \beta \), and the fraction of delegating investors, \( \lambda \)) and the intermediary’s preferences. When the cost of effort increases, the intermediary chooses a lower action. When the importance of the preferences increases, the intermediary responds less to the cash flow incentives and more to their preferences. This could increase or decrease the chosen action.
Note that expected cash flows are

\[
E [x] = \beta a_f = \beta (m_i + \mu_f) = \frac{\beta^2 \lambda + \beta (c \mu_f + \gamma A_{i,f})}{c + \gamma}.
\]

So, the price can also be written as \( p_f = E [x] - \tau \sigma_f^2 \). We summarize our characterization in the following Proposition.

**Proposition 1** With an interested intermediary:

1. the market-clearing price is \( p_f^* = \frac{\beta^2 \lambda + \beta (c \mu_f + \gamma A_{i,f})}{c + \gamma} - \tau \sigma_f^2 \),

2. the influence action taken by the intermediary is \( m_i^* = \frac{\beta \lambda + \gamma (A_{i,f} - \mu_f)}{c + \gamma} \),

3. the manager’s action is \( a_f^* = \frac{\beta \lambda + (c \mu_f + \gamma A_{i,f})}{c + \gamma} \),

4. and all investors hold the same portfolios regardless of delegation, i.e., \( q_j^* = q_i^* = 1 \).

We next discuss how the economic features of the setting affect the firm’s stock price, using comparative statics.

**Corollary 1** The firm’s stock price is decreasing in investor risk aversion, \( \tau \), and the variance of cash flows, \( \sigma_f^2 \). Stock price is increasing in the degree to which the manager’s action affects cash flows, \( \beta \), the fraction of delegating investors, \( \lambda \), and the manager and intermediary’s desired actions, \( \mu_f \) and \( A_{i,f} \), respectively.

As in standard one-period market models with rational risk-averse investors, the price is decreasing in \( \tau \) and \( \sigma_f^2 \). For the effects of the other parameters on price, we focus on their effects on expected cash flows. Expected cash flows are increasing in the effect of the manager’s action on cash flows, \( \beta \), the fraction of delegating investors, \( \lambda \), the manager’s desired action, \( \mu_f \), and the intermediary’s desired action, \( A_{i,f} \). An increase in \( \beta \) increases the benefit of inducing the manager to exert cash flow-increasing effort (for \( \beta > 0 \)). A greater
fraction of delegating investors causes the intermediary to care more about increasing cash flows via influence, leading to more influence efforts and greater cash flows. An increase in $A_{i,f}$ also leads to higher expected cash flows, as long as $\gamma > 0$, because it induces the intermediary to choose more positive influence. A higher value for $\mu_f$ increases expected cash flows because it implies the manager, all else equal, prefers a higher level of productive effort.

**Corollary 2** The firm’s stock price can be increasing or decreasing in the cost terms related to influence, $\gamma$ and $c$. Price is increasing in the cost of influence effort, $c$, if $\gamma (A_{i,f} - \mu_f) + \beta \lambda < 0$ and decreasing in $c$ otherwise. Price is increasing in the cost of the manager’s action deviating from the intermediary’s interests, $\gamma$, if $c (A_{i,f} - \mu_f) - \beta \lambda > 0$, and decreasing otherwise.

Interestingly, expected cash flows and price can be increasing or decreasing in $c$ and $\gamma$, the cost terms in the intermediary’s objective function related to influence. The signs of the effects of $c$ and $\gamma$ on expected cash flows depend on the signs of $-\gamma (A_{i,f} - \mu_f) - \beta \lambda$ and $c (A_{i,f} - \mu_f) - \beta \lambda$, respectively, as

$$\frac{dp_f}{dc} = \frac{d}{dc} \left( \frac{\beta^2 \lambda + \beta (c \mu_f + \gamma A_{i,f})}{c + \gamma} - \tau \sigma_f^2 \right) = \frac{\beta (A_{i,f} - \mu_f) - \beta \lambda}{(c + \gamma)^2},$$

and

$$\frac{dp_f}{d\gamma} = \frac{d}{d\gamma} \left( \frac{\beta^2 \lambda + \beta (c \mu_f + \gamma A_{i,f})}{c + \gamma} - \tau \sigma_f^2 \right) = \frac{\beta c (A_{i,f} - \mu_f) - \beta \lambda}{(c + \gamma)^2}.$$

When $\gamma (A_{i,f} - \mu_f) + \beta \lambda > 0$, $\frac{dp_f}{dc} < 0$, because a higher cost of influence causes the intermediary to choose a lower level of influence. However, increasing $c$ increases cash flows when $\gamma (A_{i,f} - \mu_f) + \beta \lambda < 0$, which requires $A_{i,f} - \mu_f < 0$. In this scenario, the intermediary’s preferences cause it to exert negative influence efforts, $m_i^* < 0$, which lower the manager’s effort and, thus, lower cash flows. In such a case, an increase in the cost of monitoring causes the intermediary to reduce the magnitude of influence, which implies moving $m_i^*$ closer to zero by increasing it.
For the effects of \( \gamma \), \( c (A_{i,f} - \mu_f) - \beta \lambda > 0 \) is equivalent to \( A_{i,f} - \mu_f > \beta \lambda / c \), which implies the intermediary’s preferred action is higher than the manager’s. An increase in \( \gamma \) causes the intermediary to care more about this divergence, which results in a more positive influence effort, an increase in the manager’s action, and higher cash flows and price. In contrast, if \( c (A_{i,f} - \mu_f) - \beta \lambda < 0 \iff A_{i,f} - \mu_f < \beta \lambda / c \), an increase in \( \gamma \) will result in a lower stock price because for this range of parameters an increase in \( \gamma \) causes the intermediary to choose a more negative level of influence. When \( A_{i,f} < \mu_f \), this is straightforward, as the intermediary prefers for the manager to take a lower action. For \( A_{i,f} \in (\mu_f, \mu_f + \beta \lambda / c) \), the intermediary’s private preference for managerial action is greater than the manager’s. However, it remains lower than the action the manager would take in the presence of an intermediary who has no private preference (\( \gamma = 0 \)) but is otherwise equivalent. That is, the intermediary values the effect of influence on expected cash flows, which pushes the manager’s optimal action up to \( \mu_f + \beta \lambda / c \). However, this is higher than the intermediary’s private preference, so an increase in \( \gamma \) again causes the intermediary to exert a more negative influence effort.

Now, assume that investors who delegate to the intermediary must compensate the intermediary for her efforts (e.g., via asset management fees). Technically, this is similar to bringing in a participation constraint for the intermediary. With such fees equally split across delegating investors, we have

\[
CE_i^* = \frac{CE_i}{\lambda} = \frac{1}{2} \tau \sigma_f^2 - \frac{1}{2} \frac{\beta^2 \lambda^2 + c \gamma (\mu_f - A_{i,f})^2}{\lambda (c + \gamma)}. \tag{11}
\]

For direct investors, the equilibrium certainty-equivalent is

\[
CE_j^* = \frac{1}{2} \tau \sigma_f^2. \tag{12}
\]

Note that \( CE_j^* \) is the same as in a model without influence. The reason is that price captures all first-moment effects and the expected utility solely derives from the second moment, which is not affected by influence in our model. Clearly, a comparison of (11) and
(12) yields $CE_i^* \leq CE_j^*$. This should be apparent absent calculation as $CE_i^* = CE_j^* - \left(\gamma (a_f - A_{i,f})^2 + cm_i^2\right)/(2\lambda) \leq CE_j^*$, with equality only if $(\gamma (a_f - A_{i,f})^2 + cm_i^2)/(2\lambda) = 0$.

**Lemma 2** Absent additional frictions, delegating investors are worse off in expectation than direct investors.

The ordering of certainty equivalents between delegating and direct investors, $CE_i^* < CE_j^*$, implies that each investor is better off investing directly rather than through the intermediary. The intermediary’s actions are foreseeable for a given $\lambda$ and are thus impounded into price in such a way as to stop the intermediary (and the delegating investors) from ex ante benefitting from the ability to take them. Furthermore, investors hold the same portfolios regardless of whether they invest directly or through the intermediary.

**Proposition 2** Absent additional frictions, no investors would delegate to the intermediary.

Earlier, we highlighted the free-rider problem that caused influence efforts to go to zero in the setting without the intermediary. From an atomistic investor’s perspective, influence is materially costly, but the benefits to a single atomistic investor are negligible despite aggregate benefits. The intermediary can act on behalf of a set of investors, thus overcoming the free-riding problem amongst the delegating investors. However, our results thus far show that a free-rider problem remains. The direct investors benefit from the intermediary’s efforts and, because they invest directly, can avoid having to pay the intermediary. This free-rider problem eliminates the benefit from delegating investment. This result stands in contrast to the literature on blockholder monitoring that suggests that blockholders overcome the free-rider problem of monitoring. Our analysis points out that, in effect, one free-rider problem merely gets substituted for another. In the next section, we introduce a cost to direct investment, as in Admati et al. (1994), to arrive at an interior level of equilibrium delegation and examine the effects of endogenous delegation on our previously derived results.
We now assume the direct investors pay a (net) transaction cost of $\kappa$ to participate in the market. As in Admati et al. (1994), the intermediary also bears a cost $\kappa$, but this can be viewed as negligible when spread across any measurable mass of delegating investors. The certainty-equivalent utilities are given by $CE_i^*$ and $CE_j^* - \kappa$, where $CE_i^*$ and $CE_j^*$ are defined in (11) and (12), respectively. Before solving for the optimal fraction of delegating investors, $\lambda$, we analyze the certainty equivalent of a delegating investor. First, note that his expected utility is decreasing in the preference disalignment between the intermediary and the manager, $(\mu_f - A_{i,f})^2$,

$$\frac{dCE_i^*}{d(\mu_f - A_{i,f})^2} = -\frac{c\gamma}{2\lambda^2(c+\gamma)} < 0.$$  

This happens because a greater preference disalignment causes the intermediary to choose a higher influence action. Because the cash-flow effects are priced in, this higher actions is costly to the intermediary and, therefore, to the delegating investors. As a result, the intermediary (and delegating investors) is better off when the fund manager’s preferences align with those of the corporate manager, i.e., when $A_{i,f} \rightarrow \mu_f$. Furthermore, note that the expected utility of delegating investors can be increasing or decreasing in $\lambda$, as

$$\frac{dCE_i^*}{d\lambda} = \frac{d}{d\lambda} \left( -\frac{1}{2} \frac{\beta^2 \lambda^2 + c\gamma (\mu_f - A_{i,f})^2}{\lambda (c+\gamma)} \right) \propto c\gamma (\mu_f - A_{i,f})^2 - \beta^2 \lambda^2.$$  

In isolation, the second term causes the expected utility of delegating investors to decrease in the fraction of delegating investors. When more investors delegate, the intermediary will choose a higher action (to increase cash flows), which reduces expected utilities. However, the first-term has an opposing effect. When more investors delegate, the intermediary puts a lower relative weight on their own preferences. This works to reduce the intermediary’s action, which increases expected utility. The size of these two effects determines the total
Corollary 3 All else equal, the intermediary and delegating investors are better off when the fund manager’s preferences are more closely aligned with the manager’s, i.e., when $(\mu_f - A_{i,f})^2$ is small. However, an increase in the cost of preference misalignment between the intermediary and manager, $\gamma$, can make the intermediary and delegating investors better or worse off.

Many types of funds operate without such preferences over managerial actions above and beyond cash flows. However, they can be beneficial in equilibrium if $\frac{dCE^*_{i}}{d\gamma} > 0$. Corollary 3 notes this possibility, which can be seen from

$$
\frac{dCE^*_{i}}{d\gamma} = \frac{d}{d\gamma} \left( -\frac{1}{2} \frac{\beta^2 \lambda^2 + c \gamma (\mu_f - A_{i,f})^2}{\lambda(c + \gamma)} \right) = -\frac{1}{2} \frac{c^2 (\mu_f - A_{i,f})^2 - (\beta \lambda)^2}{\lambda(c + \gamma)^2} \propto (\beta \lambda)^2 - c^2 (A_{i,f} - \mu_f)^2 \geq 0.
$$

Interestingly, the intermediary’s certainty equivalent is increasing in $\gamma$ if the cost of influence is small (low $c$), managerial actions have a large effect on expected cash flows (high $\beta$), or the intermediary and manager have aligned preferences for action ($A_{i,f}$ is close to $\mu_f$). In these cases, the optimal influence action is high and positive, governed primarily by the effect of influence on expected cash flows via the manager’s effort. However, as Lemma 2 establishes, delegating investors suffer from the intermediary’s effort because while the positive effect on cash flows is priced, they have to bear the cost of effort. Increasing $\gamma$ effectively commits the intermediary to exerting less of the costly effort, as it causes the intermediary to place more weight on the action-preference divergence, $(a_f - A_{i,f})^2$. In turn, this increased weight is harmful when $(A_{i,f} - \mu_f)^2$ is high, as it then causes the intermediary to increase the magnitude of influence efforts. Overall, our finding that the intermediary can benefit from high $\gamma$ when preferences are aligned provides a novel justification for the joint occurrence of...
intermediaries being greatly concerned about corporate governance (McCahery et al., 2016) while frequently voting with management (e.g., Heath et al., 2019).

Now, assume that investors choose in an initial period \( t = 0 \) prior to the trading round whether to delegate to the intermediary or invest directly. The remainder of the game is unchanged. This merely serves to endogenize the fraction of delegating investors. In equilibrium, we will have this the equilibrium \( \lambda^* \) defined by \( CE_i^* = CE_j^* - \kappa \), which we can write as

\[
\lambda^* = \frac{1}{\beta^2} \left( \kappa (c + \gamma) + \sqrt{(\kappa (c + \gamma))^2 - \beta^2 c \gamma (\mu_f - A_{i,f})^2} \right).
\]

The solution is characterized in the following proposition.

**Proposition 3** When direct investors incur a fixed cost of \( \kappa \), the fraction of investors who delegate is given by

\[
\lambda^* = \frac{1}{\beta^2} \left( \kappa (c + \gamma) + \sqrt{(\kappa (c + \gamma))^2 - \beta^2 c \gamma (\mu_f - A_{i,f})^2} \right).
\]

The fraction of investors who delegate is decreasing in the effect of managerial action on cash flows, \( \beta \), and the divergence between the manager and intermediary’s preferences, \( (\mu_f - A_{i,f})^2 \), and independent of investors’ risk aversion, \( \tau \), and the variance of cash flows, \( \sigma_f^2 \).

The equilibrium fraction of delegating investors is decreasing in the cash-flow efficiency of managerial effort, \( \beta \), and the divergence between the intermediary and manager’s privately preferred actions, \( (\mu_f - A_{i,f})^2 \). When these are larger, the intermediary will exert more influence effort, which increases the costs that are borne by the delegating investors and reduces the benefit of delegation relative to direct investing.

Interestingly, the effects of increases in the cost parameters on equilibrium delegation, \( \frac{d\lambda^*}{dc} \) and \( \frac{d\lambda^*}{d\tau} \), are non-monotonic. As shown above, increases in these can increase or decrease the intermediary’s equilibrium influence efforts, which leads to non-monotonic effects on prices (see Corollary 2).
Based on our assumptions thus far, the optimal fraction of delegating investors is independent of investors’ risk aversion, $\tau$, and the variance of cash flows, $\sigma_f^2$. This should not be surprising, as investors are homogenous when it comes to risk tolerance and their share holdings are in equilibrium independent of the delegation decision. Later, we consider alternative modeling choices in which information asymmetry or the potential for direct investors to make sub-optimal trading decisions affect the expected payoffs to direct and delegating investors. Under these alternatives, risk tolerance and the variance of cash flows can have nontrivial effects on the fraction of investors who delegate.

**Corollary 4** The stock price, $p_f^*$, managerial action, $a_f^*$, and intermediary’s influence, $m_i^*$, are increasing in the cost of direct investing, $\kappa$.

Corollary 2, which results from applying the chain rule to the earlier results regarding the effects of $\lambda$, implies that the stock price is increasing in the cost of investing directly. This occurs because an increase in the fraction of delegating investors causes the intermediary to care more about the effect of influence on cash flows relative to the intermediary’s private preference, $A_{i,f}$. Influence efforts thus become more positive, increasing expected cash flows. Because this effect can be entirely anticipated, it is reflected in the market-clearing price. This result provides an interesting price benefit to costs of direct investing. Note that these are best interpreted as net costs (i.e., costs of direct investing above and beyond costs of investing through the intermediary), so our result does not suggest that atomistic investors should be prevented from participated in the market, only that there is a benefit from encouraging investors to invest through intermediaries.

### 3 Extensions

This section provides results related to several extensions of the main model.
3.1 Interested investors

In this extension, we assume that investors are affected by the actions managers take. Let

\[ CE_{j,II} = q_j (\beta a_f - p_f) - \frac{1}{2} \tau j^2 \sigma_f^2 - \frac{\gamma}{2} (a_f - A_{j,f})^2 - \frac{c}{2} m_j^2 \]  

(14)

and take the fraction of delegating investors, \( \lambda \), as exogenous. The manager’s choice of action is given by

\[
a_f^* (m_i, \{ m_j \}) \in \arg \max_{a_f} - \left( a_f - m_i - \mu_f - \int_{j=(1-\lambda)}^{1} m_j dj \right)^2
\]

\[
\Rightarrow a_f^* (m_i, \{ m_j \}) = m_i + \mu_f + \int_{j=(1-\lambda)}^{1} m_j dj
\]

Despite their interest in the manager’s action, the optimal direct-investor influence continues to be zero. As before, the effect on managerial effort is negligible (i.e., \( \frac{d}{dm_j} a_f^* (m_i, \{ m_j \}) = dj \approx 0 \)), while the private costs are not (i.e., \( cm_j^2 / 2 > 0 \) for \( m_j > 0 \)). The manager’s optimal effort therefore remains \( a_f^* = m_i + \mu_f \). Even though investors have preferences over the managers’ actions that do not depend on cash flows or their holdings, \( q_j \), the free riding problem on influence continues to prevent direct investors from causing the firm’s manager to impound those preferences into her action choice. Furthermore, this occurs even when direct investors are in complete agreement with respect to their privately preferred action, \( A_{j,f} \).

The intermediary’s objective is

\[ CE_{i,II} = \lambda \left( q_i (\beta a_f - p_f) - \frac{1}{2} \tau i^2 \sigma_f^2 - \frac{\gamma}{2} (a_f - A_{i,f})^2 \right) - \frac{\gamma}{2} (a_f - A_{i,f})^2 - \frac{c}{2} m_i^2 \]  

(15)

such that the intermediary internalizes the delegating investors’ preferences. The optimal
Influence effort is given by \( m_{i,II}(q_i) \in \text{arg max}_{m_i} CE_{i,II} \) as

\[
m_{i,II}(q_i) = \frac{\gamma(A_{i,f} - \mu_f) + \lambda \gamma (A_{j,f} - \mu_f) + \beta \lambda q_i}{(c + \gamma + \lambda \gamma)}.
\]  

(16)

Substitute this into the expression for \( CE_{i,II} \) in (15), then maximize over \( q_i \) to obtain

\[
q_{i,II}^*(p_f) = \frac{\beta (c \mu_f + \gamma A_{i,f} + \lambda \gamma A_{j,f}) - p_f (c + \gamma + \lambda \gamma)}{\tau \sigma_f^2 (c + \gamma + \lambda \gamma) - \beta^2 \lambda},
\]

which implies \( q_{i,II}^*(p_f) = q_{j,II}^*(p_f) \), as in the setting in which direct investors cared only about cash flows.

Market clearing defines price as \( 1 = \lambda q_{i,II}^*(p_f) + (1 - \lambda) q_{j,II}^*(p_f) \), yielding

\[
p_{f,II} = \frac{\beta (c \mu_f + \gamma A_{i,f} + \lambda \gamma A_{j,f} + \beta \lambda)}{c + \gamma + \lambda \gamma} - \tau \sigma_f^2.
\]

Substituting the equilibrium price and quantities into the expression for \( m_{i,II}^* \) above results in the equilibrium influence effort and managerial action of

\[
m_{i,II}^* = \frac{\beta \lambda + \gamma (A_{i,f} - \mu_f) + \lambda \gamma (A_{j,f} - \mu_f)}{c + \gamma + \lambda \gamma}, \quad \text{and}
\]

\[
a_{f,II}^* = \frac{c \mu_f + \gamma A_{i,f} + \lambda \gamma A_{j,f} + \beta \lambda}{c + \gamma + \lambda \gamma}.
\]

(17)
As before, the market-clearing price is equal to expected cash flow, $\beta a_{f,II}^*$ net of the risk premium. Differentiating the $p_{f,II}^*$ and $a_{f,II}^*$ with respect to $A_{j,f}$ and $\lambda$ yields the following result.

**Proposition 4** When investors are interested in the manager’s action above and beyond cash flows, the effect of investors’ preference parameter, $A_{j,f}$, on managerial actions, $a_f$, and stock price, $p_f$, is increasing in the fraction of investors who delegate, $\lambda$.

The result in Proposition 4 is a consequence of the fact that intermediaries exert influence effort on behalf of their portfolio investors, while direct investors find it optimal to exert no influence efforts. A greater fraction of delegating investors increases the weight that the intermediary places on investors’ preferences relative to the intermediary’s preferences and the cost of influence effort. This result is interesting because it implies a potentially counterintuitive result: direct investors’ preferences may be better reflected if direct investors play a weaker role in the market, i.e., as $\lambda$ increases.

We next turn to the effects of investors’ preferences on the optimal fraction of delegating investors. The expected utility of a delegating investor, as above, can be written as

$$
\frac{CE_{i,II}^*}{\lambda} = CE_{j,II}^* - \left( \frac{1}{2\lambda} \gamma \left( a_{f,II}^* - A_{i,f} \right)^2 + c \left( m_{i,II}^* \right)^2 \right).
$$

The potential for influence makes delegating investors worse off, as in the main model.

If direct investors bear a fixed cost, $\kappa$, of participating directly in the stock market, equilibrium $\lambda$ will be defined by

$$
\kappa = \frac{1}{2\lambda} \left( \gamma \left( a_{f,II}^* - A_{i,f} \right)^2 + c \left( m_{i,II}^* \right)^2 \right)
$$

$$
2\lambda \kappa = \gamma \left( \frac{c\mu_f + \gamma A_{i,f} + \lambda \gamma A_{j,f} + \beta \lambda}{c + \gamma + \lambda \gamma} - A_{i,f} \right)^2 + c \left( \frac{\beta \lambda + \gamma (A_{i,f} - \mu_f) + \lambda \gamma (A_{j,f} - \mu_f)}{c + \gamma + \lambda \gamma} \right)^2
$$

**Proposition 5** When investors care about the manager’s action, above and beyond its effect on the firm’s cash flows, the fraction of investors who delegate, $\lambda$, can be increasing or
decreasing in the direct investor’s preferred action, $A_{j, f}$. If $A_{j, f} < \frac{\omega \mu_f + \gamma^2 A_{h, f} - \theta \beta - \beta \gamma}{\gamma (c + \gamma)}$, then the fraction of investors who delegate is increasing in $A_{j, f}$. Otherwise, it is decreasing in $A_{j, f}$.

Proposition 5 provides the result that the fraction of investors who delegate, $\lambda$, is increasing in investors’ preference parameter, $A_{j, f}$, when $A_{j, f}$ is low and decreasing in $A_{j, f}$ when $A_{j, f}$ is high. There are two effects of an increase in $A_{j, f}$. The first is that a higher $A_{j, f}$ increases the effect of $\lambda$ on the manager’s action (see Proposition 4). The second is that a higher $A_{j, f}$ causes the intermediary to choose a larger level of costly effort, all else equal. Conversely, when $A_{j, f}$ is low or less than $\mu_f$, an increase in $A_{j, f}$ will push $A_{j, f} - \mu_f$ towards zero, which will tend to reduce the magnitude of costly influence effort provided by the intermediary, lowering the costs borne by the delegating investors.

### 3.2 An interested insider

In this extension, we introduce an interested insider into the base model. The insider has a fixed, measurable, endowment of shares, $q_h$, and does not participate in the stock market.\(^5\)

The insider can, however, exert influence efforts. Let the insider’s certainty-equivalent utility be defined as

$$CE_h = q_h (\beta a_f) - \frac{\omega}{2} q_h^2 \sigma_f^2 - \frac{\gamma}{2} (a_f - A_{h, f})^2 - \frac{c}{2} m_h^2.$$  

The risk aversion term, $\frac{1}{2} \tau q_h^2 \sigma_f^2$, will not be affected by any actions the insider takes, but her influence efforts and utility will depend on her share endowment, $q_h$, and preference parameter, $A_{h, f}$.

**Lemma 3** In the presence of an interested insider who holds $q_h$ shares and prefers for the

\(^5\) An alternative interpretation is that the “insider” has a fixed endowment of shares because she represents a passive index-tracking fund whose stock ownership is determined by the weight of the firm in the index.
manager to take action $A_{h,f}$, the optimal influence efforts conditional on holdings are

$$m^*_{h,IN}(q_i, q_h) = \frac{\gamma^2 (A_{h,f} - A_{i,f}) + c\gamma (A_{h,f} - \mu_f) + \beta (q_h (c + \gamma) - \lambda q_i)}{2c\gamma + c^2}, \text{ and}$$

$$m^*_{i,IN}(q_i, q_h) = \frac{\gamma^2 (A_{i,f} - A_{h,f}) + c\gamma (A_{i,f} - \mu_f) + \beta (\lambda q_i (c + \gamma) - \gamma q_h)}{2c\gamma + c^2}.$$

The influence effort taken by the insider is increasing in $A_{h,f}$ and her shareholdings, $q_h$, but decreasing in the intermediary’s preference parameter, $A_{i,f}$ and shareholdings, $q_i$. Similarly, the insider’s influence effort is decreasing in the insider’s preference parameter and shareholdings. These relations hold because the insider and intermediary can rationally anticipate how each others’ holdings and preferences will affect the influence efforts both take. Because the manager’s action is the sum of the influence efforts and her preference parameter, $\mu_f$, the influence efforts are strategic substitutes. A higher amount of influence undertaken by one party reduces the amount needed to be provided by the other party.

Note, too, that the influence undertaken by the insider does not disappear if her holdings go to zero. Rather, $m^*_{h,IN}(q_i, q_h = 0)$ also requires $A_{h,f} = \frac{1}{c+\gamma} (c\mu_f + \gamma A_{i,f} + \beta \lambda q_i)$, which will not generally be satisfied.

The optimal action, conditional on shares held, is

$$a^*_f(q_i, q_h) = \frac{c\mu_f + \gamma A_{h,f} + \gamma A_{i,f} + \beta q_h + \beta \lambda q_i}{c + 2\gamma}.$$

This is increasing in the intermediary and insiders’ preference parameters, in the shares they hold, and in the fraction of delegating investors.

**Proposition 6** In the presence of an interested insider who holds $q_h$ shares:
1. the intermediary optimally holds

\[ q_{i,IN}^* (q_h) = \frac{\beta \gamma^2 (1 - \lambda) (c (\mu_f - A_{i,f}) + \gamma (A_{h,f} - A_{i,f})) + c \tau \sigma_f^2 (c + 2 \gamma)^2}{c \tau \sigma_f^2 (c + 2 \gamma)^2 + \beta^2 \lambda \gamma (1 - \lambda) (c + \gamma)} \]

\[ + \frac{(\beta^2 \gamma^2 (1 - \lambda) - c \tau \sigma_f^2 (c + 2 \gamma)^2) q_h}{c \tau \sigma_f^2 (c + 2 \gamma)^2 + \beta^2 \lambda \gamma (1 - \lambda) (c + \gamma)}, \]

2. direct investors optimally hold

\[ q_{j,IN}^* (q_h) = \frac{\beta \lambda \gamma^2 (c (A_{i,f} - \mu_f) + \gamma (A_{i,f} - A_{h,f})) + c \tau \sigma_f^2 (c + 2 \gamma)^2 + \beta^2 \lambda \gamma (c + \gamma)}{c \tau \sigma_f^2 (c + 2 \gamma)^2 + \beta^2 \lambda \gamma (1 - \lambda) (c + \gamma)} \]

\[ - \frac{(c + 2 \gamma) (\beta^2 \lambda \gamma + c \tau \sigma_f^2 (c + 2 \gamma)) q_h}{c \tau \sigma_f^2 (c + 2 \gamma)^2 + \beta^2 \lambda \gamma (1 - \lambda) (c + \gamma)}, \text{ and} \]

3. the stock price is

\[ p_{f,IN}^* = \left( \begin{array}{c} c \beta (\beta^2 \lambda \gamma (1 - \lambda) + \tau \sigma_f^2 (2c \gamma + \lambda \gamma^2 + c^2)) \mu_f \\ + \beta \tau \gamma \sigma_f^2 (2c \gamma - \lambda \gamma^2 + c^2 - c \lambda \gamma) A_{i,f} \\ + \beta \gamma (\beta^2 \lambda \gamma (1 - \lambda) + \tau \sigma_f^2 (2c \gamma + \lambda \gamma^2 + c^2)) A_{h,f} \\ + (c \tau^2 (c + 2 \gamma)^2 \sigma_f^4 + \beta^2 \tau (c + 2 \gamma) (c (1 - \lambda) + \lambda \gamma) \sigma_f^2 + \beta^4 \lambda \gamma (1 - \lambda)) q_h \\ - \tau \sigma_f^2 (c \tau \sigma_f^2 (c + 2 \gamma)^2 - \beta^2 \lambda (c \gamma - \gamma^2 + c^2)) \\ \end{array} \right) \]

\[ \frac{1}{c \tau \sigma_f^2 (c + 2 \gamma)^2 + \beta^2 \lambda \gamma (1 - \lambda) (c + \gamma)}. \]

What we see from the expression for \( p_{f}^* \) is that the price is increasing in the insider’s private preference, \( A_{h,f} \), and her holdings, \( q_h \). An increase in either of these leads to more positive effort from the insider, which leads to a higher action from the manager, and thus higher expected cash flows.

Focusing on the optimal share holdings, an increase in \( q_h \) is associated with lower holdings by direct investors and either higher or lower holdings by delegating investors (higher if \( \beta^2 \gamma^2 (1 - \lambda) > c \tau \sigma_f^2 (c + 2 \gamma)^2 \), and lower otherwise). For both, higher \( q_h \) implies fewer shares available, which reduces their holdings. However, for the intermediary, higher \( q_h \) also allows
the intermediary to choose a lower level of influence effort, which makes shareholdings more valuable, all else equal.

The insider’s preference, \( A_{h,f} \), tends to increase the shares held by delegating investors while decreasing the shares held by direct investors. A higher \( A_{h,f} \) allows for the intermediary to economize on its influence efforts, making holdings less costly. This tilts the shareholder base towards the intermediary and delegating investors. These effects are minimized if preferences are aligned between the intermediary and the insider, i.e., if \( A_{h,f} = A_{i,f} \).

### 3.3 Intermediary has private cash flow information

In this extension, we introduce private information available to the intermediary prior to trading. Specifically, we introduce a random shock to the intermediary’s cost of influence, such that the cost is \( \frac{c}{2} m_i^2 - y m_i \), with \( y \sim N(0, \sigma_y^2) \). Marinovic and Varas (2019) use a similar functional form to introduce uncertainty. They refer to uncertainty about the intermediary’s ability or preferences. For our purposes, uncertainty about the cost of influence efforts lead to variation in influence efforts and, through the effect on the manager’s efforts, cash flows. That is, variation in the intermediary’s cost of effort serves as a convenient means of introducing private information about the firm’s cash flows.

We assume that the intermediary observes \( y \) prior to the opening of the stock market. The intermediary’s demand quantity is therefore dependent on the realization of \( y \), while direct investors choose share demand based on their expectation of the intermediary’s efficacy, \( y \).

Solving via backward induction, as in the main model, we have \( a_f = m_i + \mu_f \). The optimal influence effort conditional on shares held is

\[
m_{i,PI}^*(q_i, y) \in \arg \max_{m_i} \lambda \left( q_i \left( \beta a_f - p_f \right) - \frac{1}{2} \tau q_i^2 \sigma_f^2 \right) - \frac{\gamma}{2} (a_f - A_{i,f})^2 - \frac{cm_i^2}{2} + y m_i
\]

which implies

\[
m_{i,PI}^*(q_i) = \frac{y + \gamma (A_{i,f} - \mu_f) + \beta \lambda q_i}{c + \gamma}.
\]
Plugging this into the intermediary’s objective and maximizing over $q_i$ yields the optimal quantity of shares demanded as a function of the stock price, $p_f$, and the intermediary’s efficacy shock, $y$:

$$q_{i,PI}^* (y, p_f) = \frac{\beta (y + c\mu_f + \gamma A_{i,f}) - p_f (c + \gamma)}{\tau \sigma_f^2 (c + \gamma) - \beta^2 \lambda}.$$  

From a direct investor’s perspective, the firm will have cash flows of

$$\beta a_f + \varepsilon_f = \beta m_i + \beta \mu_f = \beta \frac{y \tau \sigma_f^2 + \tau \sigma_f^2 (c\mu_f + \gamma A_{i,f}) - \beta \lambda p_f}{\tau \sigma_f^2 (c + \gamma) - \beta^2 \lambda}$$

which implies cash flows are normally distributed as:

$$x \sim N \left( \frac{\tau \sigma_f^2 (c\mu_f + \gamma A_{i,f}) - \beta \lambda p_f}{\tau \sigma_f^2 (c + \gamma) - \beta^2 \lambda}, \left( \frac{\beta \tau \sigma_f^2}{\tau \sigma_f^2 (c + \gamma) - \beta^2 \lambda} \right)^2 \sigma_y^2 + \sigma_f^2 \right).$$

The random shock to the intermediary’s efficacy makes cash flows more random from any direct investor’s perspective.

For simplicity, we assume that direct investors do not infer $y$ from $q_i$, which could in turn be inferred from $q_j$. Introducing noisy supply (i.e., the shares available for trade are random rather than 1) would provide a mechanism for limiting investors’ ability to infer $y$ from market-clearing. However, our focus is on how the intermediary’s private information affects quantities held and ex ante utilities of delegating and direct investors. Allowing direct investors to learn from price would reduce and, in the limit eliminate, the intermediary’s information advantage.

Direct investors’ share demand is thus $q_j = \frac{E[x] - p_f}{\tau \sigma_f^2}$, or

$$q_{j,PI}^* (p_f) = \frac{(\beta (c\mu_f + \gamma A_{i,f}) - p_f (c + \gamma)) \left( \tau \sigma_f^2 (c + \gamma) - \beta^2 \lambda \right)^2}{\left( \tau \sigma_f^2 (c + \gamma) - \beta^2 \lambda \right) \left( \left( \tau \sigma_f^2 (c + \gamma) - \beta^2 \lambda \right)^2 + \beta^2 \tau \sigma_f^2 \sigma_y^2 \right)}.$$  

Market clearing for a given $y$ implies $1 = \lambda q_i + (1 - \lambda) q_j$. Substituting $q_{i,PI}^* (p_f)$ and $q_{j,PI}^* (p_f)$
from above and rearranging yields the equilibrium price. Substituting this price into \( q_{i,PI}^* (p_f) \) and \( q_{j,PI}^* (p_f) \) yield equilibrium quantities.

**Proposition 7** When the intermediary has pre-trade private information about its efficacy, given by \( y \):

1. the market-clearing price is

\[
p_{i,PI}^* = \frac{\beta \lambda \left( \beta^4 \lambda^2 - \beta^3 c \gamma^2 \left( 2c + 2\lambda \gamma - \sigma_y^2 \right) + \tau^2 \sigma_f^2 (c + \gamma)^2 \right) y}{\left( c + \gamma \right) \left( \beta^3 \lambda^2 - \beta^2 \gamma \sigma_f^2 (c + \gamma)^2 - \beta^2 \lambda \sigma_f^2 \right) \left( 2c + 2\lambda \gamma - \sigma_y^2 \right)}
\]

2. quantity held by delegating investors is

\[
q_{i,PI}^* = \frac{\beta \left( 1 - \lambda \right) \left( \sigma_f^2 \left( c + \gamma \right) - \beta^2 \lambda \right) y + \left( \sigma_f^2 \left( c + \gamma \right) - \beta^2 \lambda \right)^2 + \beta^2 \sigma_f^2 \sigma_y^2}{\left( \sigma_f^2 \left( c + \gamma \right) - \beta^2 \lambda \right)^2 + \beta^2 \lambda \sigma_f^2 \sigma_y^2}
\]

3. and quantity held by direct investors is

\[
q_{j,PI}^* = \frac{\left( \sigma_f^2 \left( c + \gamma \right) - \beta^2 \lambda \right)^2 - y \beta \lambda \left( \sigma_f^2 \left( c + \gamma \right) - \beta^2 \lambda \right)}{\left( \sigma_f^2 \left( c + \gamma \right) - \beta^2 \lambda \right)^2 + \beta^2 \lambda \sigma_f^2 \sigma_y^2}
\]

Proposition 7 provides share prices and quantities held in equilibrium as functions of \( y \). The shares held by delegating (direct) investors are increasing (decreasing) in the intermediary’s influence efficacy, \( y \). Higher realizations of \( y \) lower the intermediary’s marginal cost of influence, causing it to demand more shares and leaving fewer shares to direct investors. Even for \( y = 0 \), though, delegating investors will tend to hold more shares than direct investors. This occurs because direct investors bear additional risk from the intermediary’s efficacy shock. Delegating investors have delegated the quantity choice to the intermediary, who knows \( y \) when choosing the demand quantity.
The extension with an intermediary endowed with private cash-flow relevant information parallels Grossman and Stiglitz (1980). In their model, investors choose whether to acquire a costly signal about the firm’s cash flows. Here, if we endogenize $\lambda$, investors would choose whether or not to delegate their portfolio choice to the intermediary. Although investors do not observe the intermediary’s private information directly, they benefit from it because it is used to inform the intermediary’s portfolio choice made on behalf of delegating investors. While Grossman and Stiglitz (1980) assumed an exogenous cost of information, the cost of making informed trading decisions in our model comes from the costly actions the intermediary will take in equilibrium. Although these costly actions can benefit cash flows, that benefit accrues to all investors, whether delegating or not. As highlighted earlier, it is only the delegating investors who bear the cost of the influence efforts.

**Corollary 5** When the intermediary has pre-trade private information about its efficacy, given by $y$, an increase in the variance of the intermediary’s efficacy, $\sigma_y^2$, leads to a decrease in the expected stock price, $E[p^*_{f,PI}]$, an increase in the intermediary’s expected share holdings, $E[q^*_{i,PI}]$, a decrease in direct investors’ expected holdings, $E[q^*_{j,PI}]$, and increases in the expected influence effort, $E[m^*_{i,PI}]$, managerial action, $E[a^*_{f,PI}]$, and cash flows, $E[\beta a^*_{f,PI}]$.

Increasing $\sigma_y^2$ means that the intermediary’s privately-observed efficacy has higher variance, increasing the risk imposed on direct investors. Direct investors react by reducing demand. Although delegating investors increase their demand in expectation, the net effect is to lower the price at which the market for the firm’s shares clears. The effects on influence effort, managerial action, and cash flows are direct results of these increasing in the intermediary’s shareholdings. Interestingly, when considering the total effect on stock price, the increase in expected cash flows does not offset the decrease in direct investor demand.
3.4 Direct investors make trading errors

To introduce an information advantage for the intermediary (without making the monitoring action uncertain from the investors’ perspective) we next assume that any investor who invests directly invests based on incorrect beliefs about $\varepsilon_f$, the noise in cash flows, driven by a signal, $y_j$, observed by each direct investor. Direct investors believe $y_j = \varepsilon_f + \varepsilon_j$ with each $\varepsilon_j$ independently and identically distributed as $\varepsilon_j \sim N \left( 0, \sigma_j^2 \right)$. In reality, $y_j$ is pure noise, i.e., $y_j = \varepsilon_j \sim N \left( 0, \sigma_j^2 \right)$. The intermediary knows that the $y_i$ are uninformative and ignores them. Direct investors know ex ante that they will react incorrectly to noisy signals, but cannot in the moment stop themselves. In some sense, they cannot tell noisy signals from informative signals, though we do not explicitly model informative signals in this subsection. See Bushee and Friedman (2016) for a model of mood-susceptible investors that is similar in spirit. The noise trader model of De Long et al. (1990) is also similar. The belief that noise represents true information captures investors overconfidence, in that the direct investors overvalue their private signals.

For the period in which investors choose whether to delegate their investment choices to the intermediary or invest directly, they anticipate receipt of the signal and their reaction to it. This means that by investing through the intermediary, they avoid an irrational investment but incur the costs of influence efforts (via a transfer to the intermediary to cover her costs). This has a flavor similar to Grossman and Stiglitz (1980), but with direct investors avoiding mistakes rather than obtaining a costly and truly informative signal on which to base rational decisions.

The intermediary’s optimal influence efforts and share demand are unchanged from the main model in Section 2. Direct investors believe they have information about $\varepsilon_f$. After incorrect inference from $y_j$, each direct investor believes the randomness in firm cash flows is distributed as

$$\varepsilon_f \sim N \left( \frac{y_j \sigma_f^2}{\sigma_f^2 + \sigma_j^2}, \frac{\sigma_f^2 \sigma_j^2}{\sigma_f^2 + \sigma_j^2} \right).$$
Anticipating the intermediary’s efforts, direct investors believe cash flows are distributed as

\[ \beta a_f + \varepsilon_f | y_j \sim N \left( \frac{\tau \sigma^2_f \left( \epsilon \mu_f + \gamma A_{i,f} \right) - \beta \lambda p_f}{\tau \sigma^2_f (c + \gamma) - \beta^2 \lambda} + y_j \frac{\sigma^2_f}{\sigma^2_f + \sigma^2_j}, \frac{\sigma^2_f \sigma^2_j}{\sigma^2_f + \sigma^2_j} \right) \].

Direct investor \( j \) thus demands \( \frac{E[x] - p_f}{\text{Var}[x]} \), which is

\[ q_{j,DE}^* (y_j, p_f) = \frac{y_j}{\tau \sigma^2_f} + \frac{\sigma^2_f}{\sigma^2_f} \frac{\beta \epsilon \mu_f + \gamma A_{i,f}}{\tau \sigma^2_f - \beta^2 \lambda} - p_f. \]

The market-clearing price is given by market clearing, or \( 1 = \lambda q_i + \int_{y_j = \lambda}^1 q_j dy_j \). By the law of large numbers, the linear terms in \( y_j \) drop out from \( \int_{y_j = \lambda}^1 q_j dy_j \), leaving price defined by

\[ 1 = \lambda \frac{\beta \epsilon \mu_f + \gamma A_{i,f}}{\tau \sigma^2_f - \beta^2 \lambda} - p_f + (1 - \lambda) \frac{\sigma^2_f}{\sigma^2_f} \frac{\beta \epsilon \mu_f + \gamma A_{i,f}}{\tau \sigma^2_f - \beta^2 \lambda} - p_f. \]

Price and demands are given in the following Proposition.

**Proposition 8** When direct investors react to noisy, idiosyncratic signals, \( y_j \sim N(0, \sigma^2_j) \), as if they are informative about cash flows,

1. price is given by

\[ p_{j,DE}^* = \beta \left( \frac{\sigma^2_f (1 - \lambda) + \sigma^2_j}{(\sigma^2_f (1 - \lambda) + \sigma^2_j)} \right) \frac{\epsilon \mu_f + \gamma A_{i,f}}{\tau \sigma^2_f - \beta^2 \lambda} - \sigma^2_j \left( \frac{\tau \sigma^2_f - \beta^2 \lambda}{\tau \sigma^2_f - \beta^2 \lambda} \right), \]

2. delegating investors hold portfolios with shares

\[ q_{i,DE}^* = \frac{\sigma^2_i}{\sigma^2_f (1 - \lambda) + \sigma^2_j}, \]

3. and a direct investor who observes \( y_j \) holds shares

\[ q_{j,DE}^* (y_j) = \frac{\sigma^2_f + \sigma^2_j}{\sigma^2_f (1 - \lambda) + \sigma^2_j} + \frac{y_j}{\tau \sigma^2_f}, \]
Note that $E[q_j^*] > q_i^*$, as $\sigma_j^2 > 0$. The direct investors demand more in expectation because they perceive lower cash flow risk. The optimal influence action and managerial actions are:

$$m_{i,DE}^* = \frac{\gamma (A_{i,f} - \mu_f)}{c + \gamma} + \frac{\beta \lambda}{c + \gamma \sigma_f^2 (1 - \lambda) + \sigma_j^2},$$

$$a_{f,DE}^* = \frac{\gamma A_{i,f} + c \mu_f}{c + \gamma} + \frac{\beta \lambda}{c + \gamma \sigma_f^2 (1 - \lambda) + \sigma_j^2}.$$

They are independent of $y_j$ because the intermediary’s demand is not affected by $y_j$, as the idiosyncratic noise on which direct investors make decisions does not affect price in equilibrium. Note that correlated errors, which we have not modeled here, would lead to aggregate demand shocks that would affect price and allocations in equilibrium.

**Corollary 6** When direct investors react to noisy, idiosyncratic signals, $y_j \sim N(0, \sigma_j^2)$, as if they are informative about cash flows, an increase in the variance of direct investors’ noise signals, $\sigma_j^2$, leads to a decrease in the expected shares held by direct investors, $E[q_{i,DE}(y_j)]$, and the stock price, $p_{f,DE}$, and increases in the shares held by delegating investors, $q_{i,DE}^*$, the intermediary’s influence effort, $m_{i,DE}^*$, managerial action, $a_{f,DE}^*$, and expected cash flows, $E[\beta a_{f,DE}^*]$.

Increasing $\sigma_j^2$ means that direct investors believe their signals about cash flows become worse. This effectively lowers their overconfidence, which causes them to reduce demand. Delegating investors end up holding more shares, which causes the intermediary to exert more influence efforts, increasing the manager’s action and expected cash flows. However, the reduction in direct investor demand lowers stock price. Unfortunately for them, overconfidence hits direct investors twice. First, they hold too many shares because they underestimate the riskiness of cash flows. Second, their demand attenuates the incentives of the intermediary to exert effort that would increase cash flows. So, in total the direct investors hold more shares of a firm with lower cash flows.
Turning to expected utilities, direct investors anticipate investing incorrectly due to their overconfidence in the signal, $y_j$ (think Odysseus tied to the mast). When choosing whether to delegate or invest directly, they view $y_j$ as a random variable to be realized in the future. Investors know that they will be susceptible to $y_j$ in the future and can use and have correct views about the randomness in cash flows. They know that they can use delegation of investment to avoid incorrect investing due to $y_j$.

**Proposition 9** When direct investors react to noisy, idiosyncratic signals, $y_j \sim N(0, \sigma_j^2)$, as if they are informative about cash flows, there exists an equilibrium $\lambda > 0$ such that a positive fraction of investors delegate in equilibrium.

Proposition 9 provides the result that corresponds to the intuition above. Investors can “buy” their way out of making mistakes, i.e., avoid trading decisions based on noise, by delegating their portfolios to a rational intermediary. The intermediary will in equilibrium exert costly influence efforts, and will thus require compensation that lowers delegating investors’ expected utility. In the next section, we consider a setting in which the intermediary

### 4 Conclusion

In this paper, we study the implications of interested intermediaries, who make portfolio decisions on behalf of their client investors but have preferences over the actions that firms take. We find that such preferences affect investor activism or influence, managerial actions, stock prices, and portfolio delegation choices. Free-riding on investor activism plays a central role, as influence activities can benefit all shareholders, but their costs are borne privately. In equilibrium, therefore, the existence of delegation requires intermediaries to have some advantage, e.g., via scale (spreading fixed costs) or information asymmetry.

Although we considered several extensions, there remain interesting avenues to pursue. We modeled an intermediary and insider acting independently, but did not allow for multiple intermediaries who interact in the market for shares as well as the market for influence. Nor
have we extended the model to multiple periods or multiple firms. With multiple firms, interested intermediaries might hold portfolios that are inefficient from a pure risk-return consideration. With multiple intermediaries, strategic interactions between intermediaries become important, and investors’ delegation choices become more complex. Focusing on delegation choices, Gârleanu and Pedersen (2018) model investor search for intermediaries who may have informational advantages. Strategic interactions between intermediaries become more complicated in a dynamic game, in which reputation can play a role and intermediaries can coordinate (e.g., Dimson et al., 2019).

A large shift in asset management has been the move from active to passive management, with huge sums now invested in index-tracking funds (Segal, 2019). While not addressed directly, there are several ways in which passive funds could be interpreted in the context of the model. First, they could be direct investors who in equilibrium do not take influence actions. Second, passive funds could be interpreted as intermediaries who have committed not to take any costly influence actions. They help delegating investors avoid the transaction costs of Section 2.5 while refraining from imposing influence costs on their clients. In contrast with these interpretations, however, even passive fund managers participate in influence actions such as proxy voting and other stewardship activities (Mallow, 2019; Novick et al., 2018). Third, passive funds could be related to the unskilled managers and “noise allocators” of Gârleanu and Pedersen (2018). Fourth, large index-tracking fund managers could be viewed as the “interested insider” of Section 3.2 who holds an exogenous endowment of shares dictated by an index. Overall, the effect of passive funds on corporate actions and prices implied by our model depends on how such passive funds are interpreted.

We find that interested intermediaries benefit from preference alignment with managers or with other insiders, because this allows them to economize on privately costly influence efforts. Greater costs to direct investing shift holdings towards intermediaries, which can lead to greater influence efforts and more positive expected firm cash flows while reducing stock

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6 The active-passive dichotomy here relates to stock-picking, rather than to influence or stewardship activities. Passive indexers can and do exert influence activities via voting and engagement.
price via the effect of costs on aggregate share demand. The effects of changes in influence costs depend on the intermediary’s interests and alignment with management. These results, and others described in more detail in the body of the manuscript, contribute broadly to literatures on asset management, investor activism, and corporate social responsibility.
5 Appendix

Proof of Proposition 3: Equation 13 has two positive solutions:

\[
\lambda^+ = \frac{\kappa (c + \gamma) + \sqrt{\left(\kappa (c + \gamma)\right)^2 - \beta^2 c^2 \gamma (\mu_f - A_i,f)^2}}{\beta^2}, \quad \text{and}
\]
\[
\lambda^- = \frac{\kappa (c + \gamma) - \sqrt{\left(\kappa (c + \gamma)\right)^2 - \beta^2 c^2 \gamma (\mu_f - A_i,f)^2}}{\beta^2}.
\]

To determine which \( \lambda \) is the appropriate equilibrium to focus on, let \( \delta_{i,f} = (A_{i,f} - \mu_{i,f}) \) and define

\[
G(\lambda; \cdot) = \lambda^2 \beta^2 - 2 \kappa (c + \gamma) \lambda^* + c \gamma \delta_{i,f}^2
\]
such that we can write equation (13) as \( G(\lambda; \cdot) = 0 \). As \( \kappa \) increases, direct investors are made worse off. Therefore, in any reasonable equilibrium, we must have \( \frac{d\lambda}{d\kappa} > 0 \), as an increase in the cost of direct investing will cause the fraction of delegating investors to increase. The implicit function theorem gives \( \frac{d\lambda}{d\kappa} = \frac{\partial G}{\partial \lambda} \cdot \frac{\partial G}{\partial \kappa} \). The partial with respect to \( \lambda \) is \( \frac{\partial G}{\partial \lambda} = 2 \lambda \beta^2 - 2 \kappa (c + \gamma) \), which is positive for \( \lambda^+ \) and negative for \( \lambda^- \). The partial with respect to \( \kappa \) is \( \frac{\partial G}{\partial \kappa} = -2 (c + \gamma) \lambda < 0 \). This means that \( \frac{d\lambda}{d\kappa} = 2 (c + \gamma) \lambda \frac{\partial G}{\partial \lambda} \), which implies \( \frac{\partial G}{\partial \lambda} < 0 \). The partial with respect to the direct investors’ preference parameter, \( \beta \), does not affect the sign of \( \lambda^* \).

Proof of Proposition 5: Rewrite equation (18) as

\[
0 = (2 \kappa \gamma^2) \lambda^3 + \left(4 \kappa \gamma (c + \gamma) - \gamma (\beta + \gamma A_j - \gamma A_{i,f})^2 - c (\beta - \gamma (\mu_f - A_{j,f}))^2\right) \lambda^2 + 2 \left(\kappa (c + \gamma)^2 - c \gamma^2 (\mu_f - A_{i,f})^2\right) \lambda - c \gamma (\mu_f - A_{i,f})^2 (c + \gamma)
\]

Let the RHS = \( G_3 (\cdot) \).

\[
\frac{\partial G_3}{\partial \lambda} = 6 \kappa \gamma^2 \lambda^2 + 2 \left[4 \kappa \gamma (c + \gamma) - \gamma (\beta + \gamma A_{j,f} - \gamma A_{i,f})^2 - c (\beta - \gamma (\mu_f - A_{j,f}))^2\right] \lambda
\]
\[
+ 2 \left(\kappa (c + \gamma)^2 - c \gamma^2 (\mu_f - A_{i,f})^2\right), \quad \text{and}
\]
\[
\frac{\partial G_3}{\partial \kappa} = 2 \gamma^2 \lambda^3 + 4 \gamma (c + \gamma) \lambda^2 + 2 (c + \gamma)^2 \lambda > 0.
\]

Note that for any reasonable equilibrium, we should have \( \frac{d\lambda}{d\kappa} > 0 \).

\[
\frac{d\lambda}{d\kappa} = -\frac{\partial G_3}{\partial \lambda} \cdot \frac{\partial G_3}{\partial \kappa} = -\frac{2 \gamma^2 \lambda^3 + 4 \gamma (c + \gamma) \lambda^2 + 2 (c + \gamma)^2 \lambda}{2 \gamma^2 \lambda^3}
\]

which implies \( \frac{\partial G_3}{\partial \lambda} < 0 \).
\( A_{j,f} \), is

\[
\frac{\partial G_3}{\partial A_{j,f}} = \frac{\partial}{\partial A_j} \left( (2\kappa \gamma^2) \lambda^3 + \left( 4\kappa \gamma (c + \gamma) - \gamma (\beta + \gamma A_{j,f} - \gamma A_{i,f})^2 - c (\beta - \gamma (\mu_f - A_{j,f}))^2 \right) \lambda^2 \right) + 2 \left( \kappa (c + \gamma)^2 - c \gamma^2 (\mu_f - A_{i,f})^2 \right) \lambda - c \gamma (\mu_f - A_{i,f})^2 (c + \gamma) \nonumber \\
= -2\lambda^2 \gamma (c \beta + \beta \gamma + \gamma (A_{j,f} - \mu_f) + \gamma^2 (A_{j,f} - A_{i,f})) (c + \gamma) 
\]

And, from the chain rule, \( \frac{\partial A_{j,f}}{\partial A_{j,f}} \propto \frac{\partial G_3}{\partial A_{j,f}} \). Note that \( \frac{\partial G_3}{\partial A_{j,f}} \geq 0 \). It is positive if

\( A_{j,f} < c \gamma \mu_f + \gamma^2 A_{i,f} - c \beta - \beta \gamma \gamma (c + \gamma) \).

So, for low \( A_{j,f} \), the fraction of investors who delegate is increasing in \( A_{j,f} \).

\textbf{Proof of Lemma 3:} Understanding that direct investors continue to exert no influence efforts, the manager’s action choice is given by

\[
a_f^* (m_i, m_h) \in \arg \max_{a_f} - \left( a_f - m_i - \mu_f - m_h \right)^2 \\
\Rightarrow a_f^* (m_i, m_h) = m_i + \mu_f + m_h. 
\]

The insider’s optimal influence effort is

\[
m_{h,IN}^* \in \arg \max_{m_h} \left( q_h (\beta a_f) f - \frac{1}{2} \tau q_h^2 \sigma_f^2 - \frac{\gamma}{2} (a_f - A_{h,f})^2 - \frac{c}{2} m_h^2 \right) \\
= \arg \max_{m_h} \left( q_h \left( \beta \left( m_i + \mu_f + m_h \right) \right) - \frac{1}{2} \tau q_h^2 \sigma_f^2 - \frac{\gamma}{2} \left( m_i + \mu_f + m_h - A_{h,f} \right)^2 - \frac{c}{2} m_h^2 \right) 
\]

with FOC

\[-(cm_h + \gamma \mu_f + \gamma m_h + \gamma m_i - \beta q_h - \gamma A_{h,f}) = 0 \]

implying

\[ m_{h,IN}^* (q_h, m_i) = \frac{\beta q_h + \gamma (A_{h,f} - \mu_f - m_i)}{c + \gamma}. \]

The intermediary’s optimal influence effort is

\[
m_{i,IN}^* \in \arg \max_{m_i} \lambda \left( q_i (\beta a_f) f - \frac{1}{2} \tau q_i^2 \sigma_f^2 \right) - \frac{\gamma}{2} (a_f - A_{i,f})^2 - \frac{c}{2} m_i^2 \\
= \arg \max_{m_i} \lambda \left( q_i \left( \beta \left( m_i + \mu_f + m_h \right) \right) - \frac{1}{2} \tau q_i^2 \sigma_f^2 \right) - \frac{\gamma}{2} \left( m_i + \mu_f + m_h - A_{i,f} \right)^2 - \frac{c}{2} m_i^2 
\]

with FOC

\[-(cm_i + \gamma \mu_f + \gamma m_h + \gamma m_i - \gamma A_{i,f} - \beta q_i) = 0 \]

implying

\[ m_{i,IN}^* (q_i, m_h) = \frac{\beta \lambda q_i + \gamma (A_{i,f} - \mu_f - m_h)}{c + \gamma}. \]
Note that the optimal influence efforts from the intermediary and insider depend on each other. We solve for the Nash equilibrium in the subgame:

\[
m_{h,IN}(q_i, q_h) = \frac{\gamma^2 (A_{h,f} - A_{i,f}) + c \gamma (A_{h,f} - \mu_f) + \beta (q_h (c + \gamma) - \lambda q_i)}{2c\gamma + c^2}, \quad \text{and}
\]

\[
m_{i,IN}(q_i, m_h) = \frac{\gamma^2 (A_{i,f} - A_{h,f}) + c \gamma (A_{i,f} - \mu_f) + \beta (\lambda q_i (c + \gamma) - \gamma q_h)}{c (c + 2\gamma)}.
\]

**Proof of Proposition 6:** The intermediary’s chooses demand as

\[
q_{i,IN}^*(p_f, q_h) \in \arg\max_{q_i} \lambda \left( q_i \left( \frac{\beta c \mu_f + \gamma A_{h,f} + \gamma A_{i,f} + \beta q_h + \beta \lambda q_i}{c + 2\gamma} - p_f \right) - \frac{1}{2} \tau q_i^2 \sigma_f^2 \right)
\]

\[
- \frac{\gamma}{2} \left( \frac{c \mu_f + \gamma A_{h,f} + \gamma A_{i,f} + \beta q_h + \beta \lambda q_i}{c + 2\gamma} - A_{i,f} \right)^2
\]

\[
- \frac{c}{2} \left( \frac{\gamma^2 (A_{i,f} - A_{h,f}) + c \gamma (A_{i,f} - \mu_f) + \beta (\lambda q_i (c + \gamma) - \gamma q_h)}{c (c + 2\gamma)} \right)^2
\]

\[
= \arg\max_{q_i} \frac{-\beta^2 \lambda (c \gamma - \gamma^2 + c^2) + c \tau \sigma_f^2 (c + 2\gamma)^2}{c (c + 2\gamma)^2} q_i^2
\]

\[
+ \lambda \frac{-c (c + 2\gamma)^2 p_f + \beta ((c + \gamma)^2 (c \mu_f + \beta q_h + \gamma A_{h,f}) + \gamma (c \gamma - \gamma^2 + c^2) A_{i,f})}{c (c + 2\gamma)^2} q_i
\]

\[
- \frac{1}{2} \gamma (c + \gamma) \left( \frac{c \mu_f - c A_{i,f} + \beta q_h + \gamma A_{h,f} - \gamma A_{i,f}}{c (c + 2\gamma)^2} \right)^2
\]

which has FOC:

\[
0 = -\lambda \frac{-\beta^2 \lambda (c \gamma - \gamma^2 + c^2) + c \tau \sigma_f^2 (c + 2\gamma)^2}{c (c + 2\gamma)^2} q_i
\]

\[
+ \lambda \frac{-c (c + 2\gamma)^2 p_f + \beta ((c + \gamma)^2 (c \mu_f + \beta q_h + \gamma A_{h,f}) + \gamma (c \gamma - \gamma^2 + c^2) A_{i,f})}{c (c + 2\gamma)^2} q_i
\]

implying

\[
q_{i,IN}^*(p_f, q_h) = \frac{\beta ((c + \gamma)^2 (c \mu_f + \beta q_h + \gamma A_{h,f}) + \gamma (c \gamma - \gamma^2 + c^2) A_{i,f}) - c (c + 2\gamma)^2 p_f}{c \tau \sigma_f^2 (c + 2\gamma)^2 - \beta^2 \lambda (c \gamma - \gamma^2 + c^2)}
\]

\[
\lim_{A_{h,f} \to \mu_f, q_h \to 0} q_{i,IN}^* = \frac{\beta ((c + \gamma)^2 (c \mu_f + \gamma \mu_f) + \gamma (c \gamma - \gamma^2 + c^2) A_{i,f}) - c (c + 2\gamma)^2 p_f}{c \tau \sigma_f^2 (c + 2\gamma)^2 - \beta^2 \lambda (c \gamma - \gamma^2 + c^2)}
\]
The quantity demanded by the direct investors is

\[ q_{j, IN}^* (p_f) \in \arg \max_{q_j} \left( \frac{\beta}{c + 2\gamma} \left( \frac{c \mu_f + \gamma A_{h,f} + \gamma A_{i,f} + \beta q_h}{c \sigma_f^2 (c + 2\gamma)^2 - \beta^2 \lambda (c \gamma - \gamma^2 + c^2)} \right) - p_f \right) \]

\[-\frac{1}{2} \tau \sigma_f^2 \]

which has FOC

\[ 0 = -\tau \sigma_f^2 q_j + \left( \frac{\beta}{c + 2\gamma} \left( -\beta \lambda \frac{c \mu_f + \beta q_h + \gamma A_{h,f} + \gamma A_{i,f}}{\tau \sigma_f^2 (c + 2\gamma)^2 - \beta^2 \lambda (c \gamma - \gamma^2 + c^2)} \right) - p_f \right) \]

that implies

\[ q_{j, IN}^* (p_f) = \frac{\beta \left((\beta^2 \lambda \gamma + c^2 \tau \sigma_f^2 + 2c \tau \gamma \sigma_f^2) \left(c \mu_f + \beta q_h + \gamma A_{h,f} \right) + (\tau c^2 \gamma \sigma_f^2 + 2 \tau c^2 \sigma_f^2) A_{i,f} \right)}{\tau \sigma_f^2 (c + 2\gamma)^2 - \beta^2 \lambda (c \gamma - \gamma^2 + c^2)} \]

\[-\frac{(\beta^2 \lambda \gamma (c + \gamma) + c \sigma_f^2 (c + 2\gamma)^2) p_f}{\tau \sigma_f^2 (c + 2\gamma)^2 - \beta^2 \lambda (c \gamma - \gamma^2 + c^2)} \cdot \]

The price is thus given by

\[ 1 - q_h = \lambda q_{i, IN}^* + (1 - \lambda) q_{j, IN}^* \]

\[ 1 - q_h = \frac{\lambda \beta \left((c + \gamma)^2 \left(c \mu_f + \beta q_h + \gamma A_{h,f} \right) + \gamma (c \gamma - \gamma^2 + c^2) A_{i,f} \right) - c (c + 2\gamma)^2 p_f}{c \sigma_f^2 (c + 2\gamma)^2 - \beta^2 \lambda (c \gamma - \gamma^2 + c^2)} \]

\[ + (1 - \lambda) \frac{- (\beta^2 \lambda \gamma (c + \gamma) + c \sigma_f^2 (c + 2\gamma)^2) p_f}{\tau \sigma_f^2 (c + 2\gamma)^2 - \beta^2 \lambda (c \gamma - \gamma^2 + c^2)} \]

\[ + (1 - \lambda) \frac{\beta \left((\beta^2 \lambda \gamma + c^2 \tau \sigma_f^2 + 2c \tau \gamma \sigma_f^2) \left(c \mu_f + \beta q_h + \gamma A_{h,f} \right) + (\tau c^2 \gamma \sigma_f^2 + 2 \tau c^2 \sigma_f^2) A_{i,f} \right)}{\tau \sigma_f^2 (c + 2\gamma)^2 - \beta^2 \lambda (c \gamma - \gamma^2 + c^2)} \]
which implies

\[
P_f = - \frac{q_h - \beta \lambda \frac{(c+\gamma)^2(c\gamma + \beta \gamma \mu_f + \sigma_f \gamma \tau_f)}{\beta^2 \lambda (c+\gamma^2 + c^2)}}{\frac{c \lambda (c+2\gamma)^2}{\beta^2 \lambda (c+\gamma^2 + c^2)} - \frac{1}{\tau \sigma_f^2} \frac{\left( e \sigma_f^2 (c+2\gamma^2 + 2\gamma^2) + (2e \gamma^2 \sigma_f^2 + c \gamma^2 \tau_f^2) \right) A_i}{\beta^2 \lambda (c+\gamma^2 + c^2)e \sigma_f^2 (c+2\gamma)^2}}}
\]

Plugging \( p_{f, IN}^* \) into the expressions for \( q_{i, IN}^* (p_f, q_h) \) and \( q_{i, IN}^* (p_f, q_h) \) yield the expressions in parts 1 and 2 of Proposition 6.

**Proof of Proposition 9:** Delegating investors have a certainty equivalent of

\[
\frac{CE_i^*}{\lambda} = \frac{q_i \left( \beta \left( \frac{\gamma (\lambda_i - \mu_f)}{c + \gamma} + \frac{\beta \lambda q_i}{c + \gamma} + \mu_f \right) \right) - p_f}{c \left( \frac{\gamma (\lambda_i - \mu_f)}{c + \gamma} + \frac{\beta \lambda q_i}{c + \gamma} + \mu_f - A_i \right)^2 - \frac{1}{2} \left( \frac{\gamma (\lambda_i - \mu_f)}{c + \gamma} + \frac{\beta \lambda q_i}{c + \gamma} + \mu_f - A_i \right)^2}
\]

At time 0 (delegation), investors view their terminal wealth as

\[
W_j = q_j (x - p) = \frac{\sigma_f^2 (\sigma_j^2 + \tau_f^2)}{\tau \sigma_j^2 (\sigma_j^2 (1 - \lambda) + \sigma_j^2)} + (\varepsilon_j + \varepsilon_f) \frac{1}{\tau \sigma_j^2} \left( \frac{\tau_f^2 \sigma_j^2}{\sigma_j^2 (1 - \lambda) + \sigma_j^2} + \varepsilon_f \right)
\]

\[
= \frac{\sigma_f^2 (\sigma_j^2 + \tau_f^2)}{\sigma_j^2 (1 - \lambda) + \sigma_j^2} + \varepsilon_j \frac{\sigma_f^2 (\sigma_j^2 + \tau_f^2)}{\tau \sigma_j^2 (\sigma_j^2 (1 - \lambda) + \sigma_j^2)} + \varepsilon_f \frac{\sigma_f^2 (\sigma_j^2 + \tau_f^2)}{\sigma_j^2 (1 - \lambda) + \sigma_j^2} + \varepsilon_j \frac{\sigma_f^2}{\sigma_j^2 (1 - \lambda) + \sigma_j^2}
\]
Let

\[
e = \left( \begin{array}{c} \varepsilon_j \\ \varepsilon_j \end{array} \right) \sim N \left( \left( \begin{array}{c} 0 \\ 0 \end{array} \right), \Sigma_e \right),
\]

where \( \Sigma_e = \left( \begin{array}{cc} \sigma^2_j & 0 \\ 0 & \sigma^2_j \end{array} \right) \),

\[
w_0 = \frac{\sigma^2_j \sigma^2_j \sigma^2_j (\sigma^2_j + \tau \sigma^2_j)}{(\sigma^2_j (1 - \lambda) + \sigma^2_j)^2},
\]

\[
w_1 = \left( \begin{array}{cc} \frac{\sigma^2_j (\sigma^2_j + \tau \sigma^2_j) + \tau \sigma^2_j \sigma^2_j}{\sigma^2_j (1 - \lambda) + \sigma^2_j} \\ \frac{\sigma^2_j (\sigma^2_j + \tau \sigma^2_j)}{\sigma^2_j (1 - \lambda) + \sigma^2_j} \end{array} \right), \text{ and}
\]

\[
w_2 = \left( \begin{array}{cc} \frac{1}{\sigma^2_j} & \frac{1}{2 \sigma_j^2} \\ \frac{1}{2 \sigma_j^2} & 0 \end{array} \right)
\]

and write expected utility as

\[
-\mathbb{E} \left[ \exp \left\{ -\tau (w_0 + w_1^T e + e^T w_2 e) \right\} \right]
\]

\[
= -|I + 2\tau w_2 \Sigma_e|^{-1/2} \exp \left\{ -\tau w_0 + \frac{1}{2} \left( -\tau \Sigma_e w_1 \right)^T (I + 2\tau w_2 \Sigma_e)^{-1} \Sigma_e^{-1} \left( -\tau \Sigma_e w_1 \right) \right\}
\]

\[
= -\frac{\sigma^2_j}{\sigma^2_j + \sigma^2_j} \exp \left\{ -\tau \left( \frac{1}{2 \tau} \sigma^2_j \sigma^2_j \tau^2 \sigma^2_j - \sigma^2_j \left( \sigma^2_j (\tau - 1)^2 - \tau^2 \sigma^2_j \right) \right) \right\}
\]

\[
= -\exp \left\{ -\tau \left( \frac{1}{2 \tau} \sigma^2_j \sigma^2_j \tau^2 \sigma^2_j - \sigma^2_j \left( \sigma^2_j (\tau - 1)^2 - \tau^2 \sigma^2_j \right) \right) - \frac{1}{2 \tau} \ln \left( \frac{\sigma^2_j}{\sigma^2_j + \sigma^2_j} \right) \right\}
\]

such that the certainty equivalent for the direct investors is

\[
CE_j = \frac{1}{2 \tau} \left( \sigma^2_j \sigma^2_j \tau^2 \sigma^2_j - \sigma^2_j \left( \sigma^2_j (\tau - 1)^2 - \tau^2 \sigma^2_j \right) \right) - \frac{1}{2 \tau} \ln \left( \frac{\sigma^2_j}{\sigma^2_j + \sigma^2_j} \right).
\]

Equating \( CE_j \) and \( CE_i / \lambda \) yields

\[
\frac{1}{2 \tau} \left( \sigma^2_j \sigma^2_j \tau^2 \sigma^2_j - \sigma^2_j \left( \sigma^2_j (\tau - 1)^2 - \tau^2 \sigma^2_j \right) \right) - \frac{1}{2 \tau} \ln \left( \frac{\sigma^2_j}{\sigma^2_j + \sigma^2_j} \right)
\]

\[
= -\frac{1}{2} c \gamma \left( \sigma^2_j (1 - \lambda) + \sigma^2_j \right)^2 \left( \mu_f - A_{i,f} \right)^2 + \lambda \sigma^2_j \left( \beta^2 \lambda - \tau \sigma^2_j (c + \gamma) \right) \left( \sigma^2_j (1 - \lambda) + \sigma^2_j \right)^2 (c + \gamma).
\]
This implies that the equilibrium $\lambda$ is defined by

\[ 0 = -\sigma_j^4 \ln \frac{\sigma_j^2}{\sigma_f^2 + \sigma_j^2} \lambda^3 + \left( 2\sigma_f^2 \left( \ln \frac{\sigma_j^2}{\sigma_f^2 + \sigma_j^2} \right) (\sigma_f^2 + \sigma_j^2) + \beta^2 \tau \frac{\sigma_f^4}{c + \gamma} + c \tau \gamma \frac{\sigma_f^4}{c + \gamma} (\mu_f - A_{i,f})^2 \right) \lambda^2 + \left( \frac{\sigma_f^2 \sigma_j^4 (\tau^2 \sigma_f^2 - \sigma_f^2 (\tau - 1)^2) + \tau^2 \sigma_j^4}{\sigma_f^2 + \sigma_j^2} - \tau^2 \sigma_f^2 \sigma_j^4 \right) \lambda + \frac{c \tau \gamma}{c + \gamma} (\sigma_f^2 + \sigma_j^2)^2 (\mu_f - A_{i,f})^2 \]

For $\lambda = 0$, the RHS is positive. For $\lambda = 1$ the RHS is

\[ \frac{\sigma_f^2}{(\sigma_f^2 + \sigma_j^2) (c + \gamma)} \left( c + \gamma \left( \ln \frac{\sigma_j^2 + \sigma_f^2}{\sigma_j^2} + \beta^2 \tau \sigma_j^2 (\sigma_j^2 + \sigma_f^2) \right) \right) \]

which may or may not be negative. As $\lambda \to \infty$, the RHS of the equation goes to $-\infty$ as the coefficient on $\lambda^3$ is negative. By continuity, it must have at least one real positive root, though this root may be greater than $\lambda$. If that is the case, then the equilibrium $\lambda$ will be defined by the constraint that $\lambda \in [0, 1]$ and be set to 1.
References


